On the efficiency of the golf swing

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A non-driven double pendulum model is used to explain the principle underlying the surprising efficiency of the golf swing. The principle can be described as a parametric energy transfer between the arms and the club head due to the changing moment of inertia of the club. The transfer is a consequence of conservation of energy and angular momentum. Because the pendulum is not driven by an external force, it shows that the golfer need do little more than accelerate the arms with the wrists cocked and let the double pendulum transfer kinetic energy to the club head. A driven double pendulum model is used to study factors affecting the efficiency of a real golf swing. It is concluded that the wrist-cock angle is the most significant efficiency-determining parameter under the golfer's control and that improvements in golf technology have had a significant impact on driving distance.
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I. INTRODUCTION

Golf is a game of frustration for players of all levels. Even the most able players experience occasions when a stroke, hit deliberately with little effort, travels further than expected. Jorgensen's[1] observations on the negative effects of wrist torque hinted at the origin of this frustration: the counterintuitive effects arising from the nonlinear physics of the golf swing. In nonlinear systems, more input does not necessarily yield more output, and so it is with the golf swing.

The double pendulum model of the golf swing was first analyzed by Williams,[2] Daish,[3] and Jorgensen.[4] These analyses and the accessible treatments in Refs. 1 and 5 have considerably advanced the understanding and coaching of golf. Subsequent analyses[6–8] have examined aspects of the swing and provided explanations of additional technique that can yield a few percent increase in distance. More recently, triple-pendulum models, which are much more realistic models of the golf swing, have been developed.[9,10] In all of these studies the nonlinearity and complexity of the model equations have obscured the basic mechanism by which the golf swing derives its efficiency.

The purpose of this paper is to demonstrate a simple mechanical model that explains the efficiency of the golf swing. The study builds on a line of thought first suggested in Refs. 3 and 11 in which the transfer of kinetic energy within the system was considered. Section II of this paper investigates the double pendulum model in which there are no external forces applied. This analysis shows that the club behaves as a type of transformer enabling the transfer of energy between the arms and the ball. The underlying energy-transfer mechanism of the golf swing is a parametric transfer dependent on the changing moment of inertia of the club head.

For typical swing parameters, the simple model predicts an optimum club-head mass near 170 g, somewhat less than typical of modern drivers. The analysis is extended in Sec. III to the standard constant-torque driven double pendulum model[1,12,13] to explain why the optimum mass is, in practice, nearer 200 g. The sensitivity of the swing efficiency and driving distance to wrist-cock angle, shaft length, shaft mass, release delay, and wrist torque is also investigated.

Section IV summarizes some of the observations and draws some conclusions. The discussion is extended to explain the energy transfer in triple-pendulum models of the swing.

II. THE NON-DRIVEN DOUBLE PENDULUM

Consider the double pendulum model in Fig. 1, which shows the golf swing reduced to its simplest elements. The mass m_1, representing the arms, is on a rigid massless arm of length L_1 that pivots about the hub. Similarly m_2 represents the club head and is on a rigid massless arm of length L_2 pivoting about m_1. To simplify the calculations we assume that both masses are point particles so that they have no moment of inertia about their centers of mass and are confined to the plane of the swing. We also assume that the ball, represented by m_3, has a coefficient of restitution of 1.0. The distance between the hub and m_2 is R, which depends on the length of the arm, the length of the club, and the wrist-cock angle, θ, according to

$$R^2 = L_1^2 + L_2^2 - 2L_1L_2 \cos \theta. \tag{1}$$

The simplest description of the model golf swing is in terms of two phases.[1,12,13] During the first phase, the arms and the club are accelerated with shoulder and wrist torques such that the wrist-cock angle remains constant. In the second phase the wrist-cock angle is no longer constrained; the club is released and allowed to swing away from the hub. During the first phase positive wrist torque is required to prevent the club from swinging into the hub. As the club accelerates, the required wrist torque decreases rapidly. The release of the club at the point where the required wrist torque is zero is described as a natural release.

We first assume that the system is not driven, that is, there are no applied torques, the gravitational potential is neglected, and the arm-club system rotates under its own inertia. Initially, both masses orbit the hub at constant angular velocity such that R is constant ($\dot{\alpha}=\dot{\beta}$), and the wrist-cock angle is fixed at its initial value $\theta=\theta_i$ as indicated in Fig. 1(a). The club is released and a short time later the mass representing the club head has swung away from the hub, as shown in Fig. 1(b), the wrist-cock angle is 180°, and the club head is about to strike the ball. To clarify the discussion we

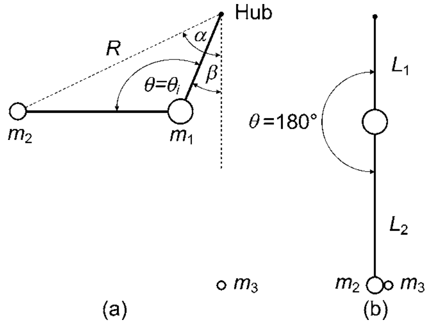

Fig. 1. The simple model of the golf swing showing the disposition of the system (a) before the club is released and (b) when the club is about to impact the ball.

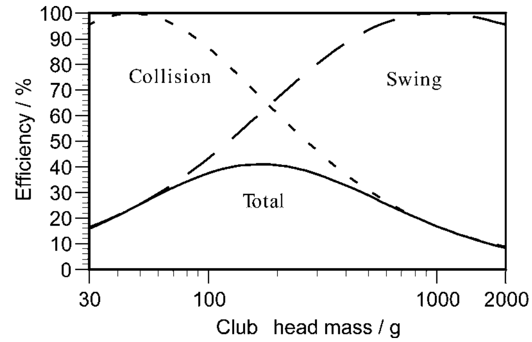

Fig. 2. Golf stroke efficiency versus club-head mass for the simple model.

shall describe the motion of the arm-club system preceding the collision with the ball as the "golf swing." We shall use the term "golf stroke" to include the swing and the subsequent club-ball collision. We define the efficiency of the swing (or the stroke) as the fraction of the total kinetic energy of the system that is present in the club head immediately before the collision (or the ball immediately after the collision).

In Figs. 1(a) and 1(b) all motion is tangential so that radial terms in the kinetic energy and angular momentum do not need to be considered. If we equate the kinetic energy T and the angular momentum L of the system in the initial and final positions

$$T=\frac{1}{2}(m_1L_1^2\dot{\beta}_i^2+m_2R^2\dot{\beta}_i^2)=\frac{1}{2}(m_1L_1^2\dot{\beta}_f^2+m_2(L_1+L_2)^2\dot{\alpha}_f^2), \tag{2}$$

$$L=m_1L_1^2\dot{\beta}_i+m_2R^2\dot{\beta}_i=m_1L_1^2\dot{\beta}_f+m_2(L_1+L_2)^2\dot{\alpha}_f, \tag{3}$$

we can determine the angular velocities of m_1 and m_2 at the instant before m_2 collides with m_3. The angles in Eqs. (2) and (3) are defined in Fig. 1 and the subscripts i and f refer to the initial and final positions, respectively.

There is one particularly interesting instance described by Eqs. (2) and (3); namely, when m_1 is stationary at the moment that the club impacts the ball. In this case all of the kinetic energy of m_1 (the arms) is transferred to m_2 (the club head) and the swing is 100% efficient. When this occurs, $\dot{\beta}_f=0$, and Eqs. (2) and (3) simplify to

$$(m_1L_1^2+m_2R^2)\dot{\beta}_i^2=m_2(L_1+L_2)^2\dot{\alpha}_f^2, \tag{4}$$

and

$$(m_1L_1^2+m_2R^2)\dot{\beta}_i=m_2(L_1+L_2)^2\dot{\alpha}_f. \tag{5}$$

The nontrivial solution of Eqs. (4) and (5) is $\dot{\beta}_i=\dot{\alpha}_f$, and therefore

$$m_1L_1^2+m_2R^2=m_2(L_1+L_2)^2. \tag{6}$$

That is, the moment of inertia of the club in the final position is the sum of the moments of inertia of the arms and the club in the initial position. This model shows that the energy (and angular momentum) of the arm-club system is redistributed purely as a consequence of the change in the moment of inertia of the club that accompanies the uncocking of the wrists. More importantly, from a coaching perspective, the system rearranges itself under its own inertia and the golfer does no work.

The condition for 100% energy transfer from the arms to the club head can be expressed, from Eqs. (1) and (6), in terms of the mass of the club head

$$m_2=\frac{m_1}{2(1+\cos\theta_i)}\frac{L_1}{L_2}. \tag{7}$$

For an arm length of 0.67 m, shaft length of 1.0 m, wrist-cock angle of 90°, and an arm mass of 3 kg, the required club-head mass is about 1.005 kg. This condition contradicts the condition for 100% energy transfer between the club and ball. Because the club-ball collision is a simple two-body collision, the condition for 100% energy transfer during the collision is that the club-head mass should equal the ball mass, that is, 46 g.

Figure 2 plots the overall efficiency for the model golf stroke versus club-head mass for a wrist-cock angle of 90° and a club-shaft length of 1.0 m. Also shown are the contributions due to the swing and the club-ball collision. The total efficiency, the product of the two dotted curves, has a broad optimum near 167 g. The solid curve, when plotted on a linear scale, is very much like that of Ref. 3, Fig. 10.5, except that the optimum in Ref. 3 is at 205 g.

The model and the swing curve of Fig. 2 show that the efficiency of the transfer from the arms to the club head varies strongly with club-head mass. However, for club-head masses near the optimum, the overall efficiency of the stroke is relatively insensitive to club-head mass. The model therefore provides a theoretical basis for the semi-empirical observations of Ref. 3.

Equation (7) also gives the optimum shaft length or wrist-cock angle, depending on the *a priori* information. For the typical club-head mass of 200 g, a 90° wrist-cock angle, and an arm length of 0.67 m, the optimum shaft length is about 5 m. If the club-head mass is 46 g, the same as the ball, then the club-ball collision is also 100% efficient, but this efficiency requires a shaft of length of about 22 m. The relatively large mass of the arms and the rules'[14] constraint on the length of the golf club to 1.219 m and the mass of the ball to 46 g, ensures that the golf stroke can never be 100% efficient.

Figure 2 also shows that the overall efficiency of the stroke can be improved if the mass given by Eq. (7) is reduced, that is, the two dotted curves in Fig. 2 are brought closer together. For example, if the shaft length is increased to 1.1 m and the wrist-cock angle reduced to 40°, then the mass of Eq. (7) is reduced to 517 g. The optimum club-head

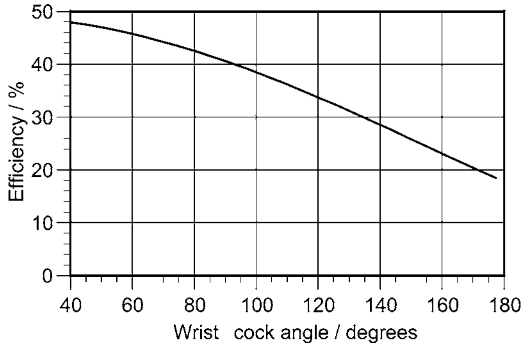

Fig. 3. Stroke efficiency versus wrist-cock angle for the simple model.

mass is then reduced to 146 g, and the overall efficiency of the model golf stroke is increased from about 41% to about 52%. As the swing efficiency increases, the optimal club-head mass decreases.

Cross[15] suggested an analogy between the golf stoke and a three-body collision. In the three-body collision a large moving mass m_1 impacts with the stationary second mass m_2, which is propelled into a collision with the third and smallest mass m_3, which is also stationary before the collision. When the first and third masses are unequal, each of the two collisions may be 100% efficient, but not at the same time. Instead there is an optimum mass between m_1 and m_3: $m_2 = (m_1 m_3)^{1/2}$.

In the model of the golf stroke the first collision is replaced by the swing and the uncocking of the wrists. Both the swing and the club-ball collision can be 100% efficient, but not at the same time within the constraints of the rules. In contrast to a three-body collision with one adjustable parameter (the mass of m_2) to optimize the energy transfer, the golf swing has three adjustable parameters, the wrist-cock angle, the club-head mass, and the shaft length.

Given the rules' constraint on the shaft length and the relative insensitivity of the efficiency to club-head mass, the wrist-cock angle is the most significant efficiency-determining factor within the golfer's control. Figure 3 plots the efficiency of the model stroke versus wrist-cock angle for a club-head mass of 200 g and a shaft length of 1.0 m. The plot clearly shows the considerable increases in the efficiency with decreasing wrist-cock angle. The efficiency for the 90° wrist-cock (the professional swing) is more than twice that for the swing with a 180° wrist-cock angle (the beginners' one-piece swing).

To complete the picture, Fig. 4 plots the efficiency versus the shaft length. As expected, the efficiency increases with increasing shaft length and exhibits a maximum near 5 m.

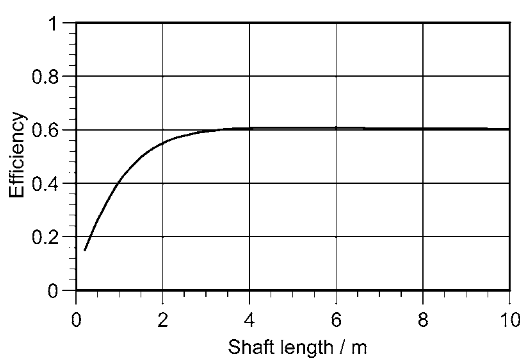

Fig. 4. Stroke efficiency versus shaft length for the simple model.

III. A MORE REALISTIC MODEL

The simple model indicates an optimum club-head mass near 170 g, but in practice, driver head masses are normally in the range 190–210 g. Additionally, the efficiency in practice is much less than the 40% value suggested by the simple model. This section discusses some of the effects responsible for the lower overall efficiency and the shift of the optimum mass to larger values.

A more realistic model was constructed as a numerical solution of the coupled second-order differential equations for a double pendulum.[1,3,13] The model omits refinements, such as the lateral shift,[1] the parametric lift,[8] or third link.[10] The two second-order differential equations were simplified to a single second-order equation using the algebraic simplifications of Pickering and Vickers,[13,16] but with the additional features of a variable wrist-cock angle and nonzero wrist torque during the second phase of the swing. As is usual, gravitational effects were neglected, and a constant torque from the shoulders and torso is assumed. The numerical solution was found using the Runge–Kutta–Fehlberg algorithm RKF45.[17,18]

The following subsections investigate the sensitivity of the swing efficiency to some of the parameters within the golfer's control. The results of the analyses are presented as perturbations with respect to a base model. The base model parameters include the following.

Arms: The arms are represented by a 3 kg point mass and an arm length of 0.67 m, as in the model in Sec. II. The resulting moment of inertia is similar to the values employed in Refs. 2, 3, and 7, and the arm length is that found by Williams[2] in the analysis of the swing of Bobby Jones.

Club: The club has a head mass of 200 g, an effective shaft length of 1.0 m, and a shaft mass of 90 g. The mass is typical of modern drivers,[19] and the shaft length is close to the effective length of a nominal 44 in. driver less approximately 120 mm to account for the length of the shaft in the golfer's hands.[1] The shaft mass is within the range used on modern graphite-shaft drivers and assumed to be uniformly distributed along the shaft. The mass of the grip is included in the mass of the arms. The club face is assumed to be rigid so that it does not provide an enhanced coefficient of restitution in the collision with the ball—the "trampoline effect."[20]

Ball: The ball has a mass of 46 g, which is close to the limit defined in the rules, and is assumed to have a coefficient of restitution of 0.78. This value is typical of USGA compliant balls[20,21] used with drivers not exploiting the trampoline effect.

Swing: The swing is assumed to have a 90° wrist-cock angle and a natural release with zero wrist torque during the second phase of the swing. The impact with the ball is assumed to take place when the club-head speed is maximum, which occurs at a wrist-cock angle of 180°. The shoulder torque is assumed to be 100 N m. This value is chosen to give realistic values for the ball velocity and kinetic energy. The value is similar to the values used in other double pendulum models,[1,2,6] but is less than the peak value of 200 N m used in Ref. 7. The value of 100 N m is considered to be large from a biomechanical perspective,[9] but is a consequence of the requirements that the double pendulum model

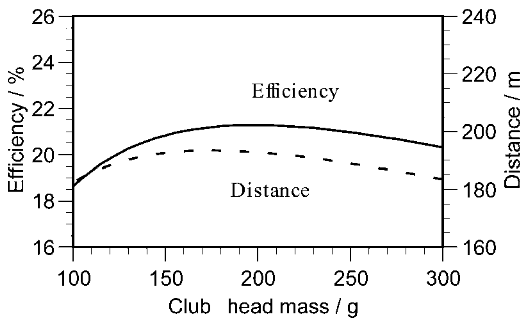

Fig. 5. Stroke efficiency and distance versus club-head mass for the driven model.

be a close fit to measured swings. In contrast, the torques required in a triple-pendulum model are within the normal capability of the body.[9]

The overall distance achieved with a given golf stroke depends on the contribution of (at least) four "figures of merit": (1) The total kinetic energy developed by the golfer. For the "natural" swing, which has zero wrist torque in the second phase, the total kinetic energy is equal to the shoulder torque times the angle traversed by the arms (downswing angle). (2) The efficiency of the swing in transferring the energy to the club head. (3) The timing, that is, the degree to which the impact with the ball occurs at the maximum club-head velocity. (4) The efficiency of the club-ball collision in transferring kinetic energy to the ball.

Because a parameter generally affects both the efficiency and the downswing angle, a more efficient swing does not necessarily result in greater distance (see the discussion of wrist torque in the following). For this reason Figs. 5–11 plot both the stroke efficiency and total distance. The stroke efficiency measures the product of the swing and collision efficiencies, and the distance is based on stroke efficiency times the total kinetic energy. Neither the effect of altered timing, the loft of the club, aerodynamics of the ball, nor fairway conditions are considered.

The distance values are calculated using the SI equivalent of the relation in Ref. 5 for the distance of a drive

$$d = 3.75 V_{\rm ball} - 25, \tag{8}$$

where $V_{\rm ball}$ is the ball velocity in m/s and the distance d is in meters. When applied to the base model, the driving distance is found to be 193 m, which is typical of a good amateur player.

Shoulder torque: Jorgensen[1] observed that the path of the club head is independent of the shoulder and wrist torques if they are scaled by the same factor. Therefore the magnitude of the shoulder torque has no impact on the efficiency derived from a model that assumes zero wrist torque. However, the presence or absence of shoulder torque is the major difference between the simple non-driven model of Sec. II and a real golf swing. During a real golf swing, the wrist-cock angle is fixed for a relatively small fraction of the swing, typically about 40% of the downswing time and about 25% of the downswing angle. During the remainder of the swing, it varies between the initial value (say 90°) and 180°. Because shoulder torque is applied throughout the swing, the efficiency with which energy is transferred to the club-head depends on a complicated weighted average of all wrist-cock angles. The result is that the efficiency of a real swing is less than implied by the simple model.

The driven double pendulum model, with a coefficient of restitution of one and zero shaft mass yields an optimum club-head mass of 193 g and an overall efficiency of about 28.7%, much less than the 41% found with the simple model. The application of torque over all wrist-cock angles therefore explains most of the loss of efficiency and increase in optimal club-head mass seen with real golf swings. A more realistic value of the coefficient of restitution of 0.78 and the nonzero shaft mass of 90 g further reduces the efficiency to about 21.3% and moves the optimum mass to 197.5 g.

Club-head mass: Figure 5 plots the stroke efficiency and distance versus the club-head mass. Note that the optima for efficiency and distance do not occur for the same clubhead mass because an increasing club-head mass reduces the downswing angle. Most significantly, both optima are close to 200 g and very broad. The impact on the driving distance of differing club-head masses in the range 155–210 g is less than 1.5 m.

These results are more qualitative than quantitative. In addition to arm mass, there are at least two effects omitted in the model that would change the optimum head mass. Muscles are able to provide greater torque at lower swing speeds,[9] and therefore a heavier club head may allow the golfer to do more work. Second, the model combines the mass of the arms and shoulders. In a triple-pendulum model, the mass representing the arms would be smaller and probably favor a lighter club head [see Eq. (7)].

Wrist-cock angle: Figure 6 shows the efficiency and distance versus wrist-cock angle, which is the most significant distinguishing feature of professional and amateur swings. For smaller wrist-cock angles, a 10° decrease in the wrist-cock angle results in approximately 8 m greater driving distance. At higher wrist-cock angles the loss in efficiency is offset by the increasing downswing angle. The downswing angles for the larger wrist-cock angles are large and anatomically impossible.

For real swings the wrist-cock angle often decreases from the initial value before the club is released. The value used in the model should be the minimum wrist-cock angle, that is, where $\dot{\theta}=0$. Among professional golfers, both the minimum and initial wrist-cock angles are often less than 90°. Williams[2] reports that Bobby Jones employed an initial wrist-cock angle of 65° and this angle appears to be common. Among current professionals, Sergio Garcia has one of the smallest minimum wrist-cock angles, estimated from photographs[22] at about 40°.

Shaft length: Figure 7 plots the efficiency and distance versus the effective shaft length. The shaft length indicated in Fig. 7 is the distance between the center of mass of the head and the axis of rotation at the wrists, which is about 120 mm less than the total shaft length.[1] The range of effective shaft lengths therefore corresponds to approximate nominal full-length shafts of between 40 and 48 in.

Figure 7 illustrates the increasing efficiency accompanying increasing shaft length, as expected from the simple model and Eq. (7). This is another case where efficiency alone is a misleading measure: increasing the length of the shaft also results in a greater downswing angle, and therefore greater total energy. The gain in distance with increasing shaft length is significant: about 11.5 m for each 100 mm increase in shaft length. Note too the key parameter in this

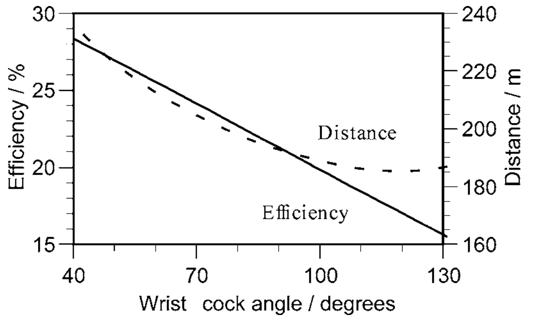

Fig. 6. Stroke efficiency and distance versus wrist-cock angle for the driven model.

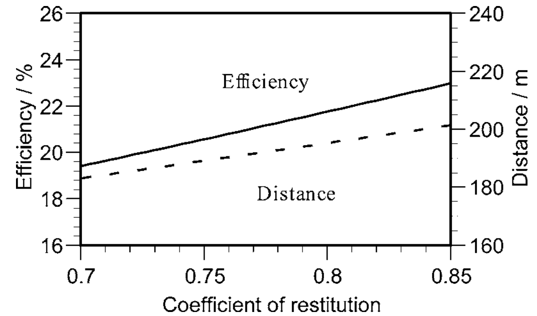

Fig. 8. Stroke efficiency and distance versus coefficient of restitution for the driven model.

analysis is not the shaft length, but the ratio of the shaft length to arm length [see Eq. (7)]. Consequently, for golfers with an arm length (strictly the radius of wrist-axis rotation about the hub) significantly less than 0.67 m, the efficiency and downswing angle will be larger. This effect may well "level the playing field" for golfers of smaller stature. In addition to the rules' constraint on shaft length, there are also anatomical limitations. With long shafts, the downswing may become anatomically impossible without the use of wrist torque.

Coefficient of restitution: The coefficient of restitution (CoR) of the club-ball collision is a significant factor in the increased driving distance that has occurred over the last three to four decades. Around 1970 the CoR was about 0.7.[1,3,5] Ball construction has since improved so that balls with a CoR of better than 0.81 are now available.[21] More recently, driver design has advanced by exploiting the trampoline effect,[20] which has enhanced the CoR for the club-ball collision to values as high as 0.86. The practical effect of the current rules is to limit the CoR of the club-ball collision to about 0.83. As Fig. 8 shows, increasing the coefficient of restitution from 0.7 to 0.83 yields about a 16 m increase in the driving distance.

Shaft mass: The primary attribute required of the shaft is tensile strength.[1,2,23] During the transfer of energy between the arms and the club head, the tension in the shaft can easily exceed 500 N. With such large forces, the flex in the shaft allows the center of mass of the driver to fall in line with that part of the shaft near the grip, resulting in an enhanced loft in the driver. Therefore, a secondary attribute of interest is the flex of the shaft—the stiffer the shaft, the less the loft enhancement.

Werner and Greig[23] explain that because of the short collision time (typically 0.5 ms) and the finite speed of transverse bending wave in the shaft, only a few centimeters of the shaft near the head are able to transfer kinetic energy to the ball. Hence, most of the kinetic energy transferred to the shaft is wasted, and we can expect a small increase in efficiency with decreasing shaft mass.

Figure 9 plots the efficiency and distance versus shaft mass. The gain in distance corresponds to about 1.5 m for each 10 g reduction in mass. Although only a minor effect, the model indicates an improvement of 12 m in driving distance between the 130 g steel shafts of the 1970s[5] and the 50 g graphite shafts now available.

Release delay: Figure 10 plots the stroke efficiency versus the release delay. The delay is expressed as the difference in downswing angle between the angle of release and the natural-release angle. Surprisingly, the swing with the natural release is the least efficient: either delaying or advancing the release results in a more efficient swing. An advanced release results in the club swinging closer to the hub, so that the wrist-cock angle decreases, and the swing becomes more efficient. When the release is delayed, the wrist-cock is maintained in the more efficient position for longer so increasing the efficiency.

As a consequence of these two effects, the overall effect of release timing is second order and has little effect on the swing efficiency. To obtain a gain of 5 m in distance, the release angle must be advanced or delayed by about 20°. Superficially, this result might appear to conflict with that of Ref. 4, which shows a significant gain for the delayed release. However the values of delay assessed in Ref. 4 were very large, above 45°, so significant gains in efficiency should be expected.

Wrist torque: Figure 11 presents the efficiency and the distance versus wrist torque, with the wrist torque given as a

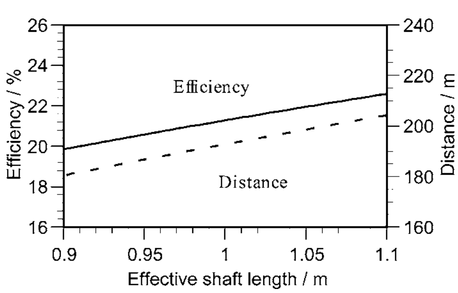

Fig. 7. Stroke efficiency and distance versus shaft length for the driven model.

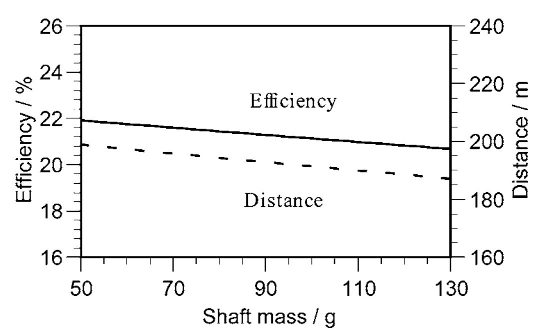

Fig. 9. Stroke efficiency and distance versus club-shaft mass for the driven model.

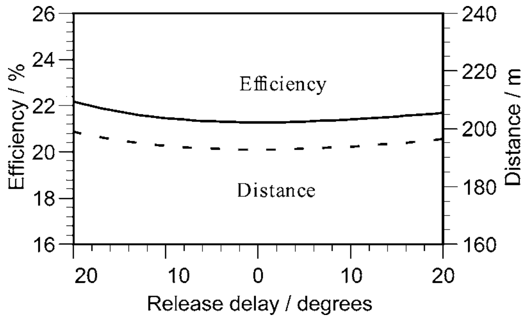

Fig. 10. Stroke efficiency and distance versus release delay for the driven model.

percentage of the shoulder torque. Simple experiments with fishing scales or weights show very quickly that human musculature limits the wrist torque to about 20% of the shoulder torque. Bobby Jones used only about 1%.[2]

One effect of the positive wrist torque is to encourage the slowing of the arms and, hence, to improve the efficiency of the swing. A second effect is to significantly reduce the downswing angle and, hence, also the kinetic energy developed by the golfer. For example, a wrist torque of 4% of the shoulder torque reduces the downswing angle by about 20°. The net effect is, as Jorgensen[1] observed, that the distance decreases with increasing wrist torque. Wrist torque has another negative effect not apparent in Fig. 11. The application of wrist torque without a corresponding reduction in the back swing will result in the club-head velocity peaking before impact, and kinetic energy being returned to the arms as the wrist-cock angle increases above 180°. Deceleration before impact is a common characteristic of amateur swings.[12]

The reduction in the downswing angle accompanying the use of wrist torque may well be useful for players with long shafted clubs (also for beginners attempting to maintain a "one-piece swing" with a large wrist-cock angle). As the shaft length increases, the total downswing angle also increases, and for golfers with short arms, natural-release swings with long-shafted drivers may be anatomically impossible. The use of wrist torque may shorten the downswing and enable the golfer to gain some benefit from the improved efficiency associated with longer shafts.

Jorgensen[4] also showed that the selective use of wrist torque late in the swing results in an increase in the driving distance. This strategy enables the golfer to gain the advantages of the wrist torque with regard to efficiency and avoid the disadvantages with regard to the shortened downswing. This principle can be applied to greater effect in triple-pendulum models, as will be explained in Sec. IV.

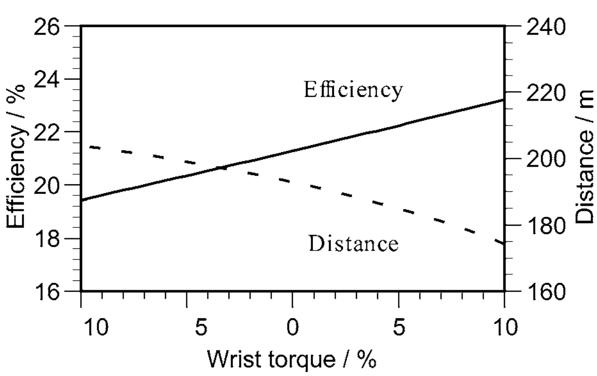

Fig. 11. Stroke efficiency and distance versus wrist torque for the driven model.

IV. CONCLUSIONS

The simple double pendulum model of Sec. II provides a physical explanation of the efficiency of the golf swing: a parametric transfer of kinetic energy accompanying the changing moment of inertia of the club as the wrists uncock. The transfer is purely a consequence of the laws of the conservation of energy and angular momentum. The fact that the model pendulum is not driven by any external force emphasizes that the golfer need do little more than accelerate the arms with the wrists cocked and then let the double pendulum transfer the energy to the club head. The model identifies the shaft length, club-head mass, and wrist-cock angle as the key parameters in the efficiency of a golf swing.

The sensitivity analysis of Sec. III provides additional insights into the golf swing. These include a physical explanation of the optimality of club-head masses near 200 g; the ineffectiveness of the release delay in improving a golf swing; the disadvantages of wrist torque; and the benefits of wrist cock. The sensitivity analysis also highlights the significant gains in driving distance that have accompanied improvements in golf technology over the last 35–40 years. For the good amateur golfer, the improvements in the CoR for the club-ball collision, the reduction in shaft mass, and a longer shaft have probably yielded a total of nearly 40 m increase in driving distance.

From a coaching perspective the model helps explain why learning a good swing can be difficult. Both the extraordinary effectiveness of wrist cock in gaining distance (without having to do additional work), and the loss in distance that occurs with the application of wrist torque are counterintuitive.

The model enables a lay description of the swing in terms of the transfer of motion with the unfolding of the arms and club. Put simply, the efficiency of the two-piece swing of the professional player, in which the arms and club unfold from a 90° wrist cock, is approximately twice that of the classic one-piece swing of the beginner. The gain in efficiency alone accounts for the 70 m increase in driving distance achieved by professionals compared to most high-handicap club players.

The underlying principle, that energy is transferred with unfolding, can be extended to explain the optimal timing of the triple-pendulum golf swing found by Sprigings and Neal (Ref. 9, Fig. 3). It also explains the comment of Turner and Hills,[10] that "… for a powerful swing the arms should stay close to the body." In the triple-pendulum model the three arms of the pendulum are comprised of the shoulders, the arms, and the club. At the beginning of the downswing, the arms are folded against the shoulders and the wrists are cocked. The swing starts with the application of torque generated by the torso to rotate the whole system. Once the system is moving quickly, torque is applied at the shoulders to unfold the arms and transfer energy from the shoulders and upper torso to the arms. Then, once the club has released naturally, wrist torque may be applied to aid the uncocking of the wrists and transfer energy from the arms to the club head. Hitting from the top, which generally employs high levels of shoulder and wrist torque early in the downswing, causes early unfolding and inhibits the efficient transfer of kinetic energy from the torso and the arms.

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