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Assessing Golfer Performance on the PGA TOUR

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The game of golf involves many different types of shots, including long tee shots (typically hit with a driver), approach shots to greens, shots from the sand, and putts on the green. Although determining the winner of a golf tournament by counting strokes is easy, assessing which factors contributed most to the victory is not. In this paper, we apply an analysis based on strokes gained, introduced previously, to assess professional golfer performance in different parts of the game [Broadie M (2008) Assessing golfer performance using Golfmetrics. Crews D, Lutz R, eds. *Sci. Golf V: Proc. World Sci. Congress Golf* (Energy in Motion, Inc., Mesa, AZ), 253–262]. Strokes gained is a simple and intuitive measure of each shot's contribution to a golfer's score and was implemented by the PGA TOUR to measure putting in May 2011. We apply strokes gained analysis to extensive ShotLink™ data to rank PGA TOUR golfers in various skill categories and to quantify the factors that differentiate these golfers. Long-game shots (those starting over 100 yards from the hole) explain about two-thirds of the score variability among PGA TOUR golfers. Tiger Woods is ranked first in total strokes gained, and at or near the top of PGA TOUR golfers in each of the three main categories: long game, short game, and putting. He dominates because he excels in all phases of the game, but his long game accounts for about two-thirds of his scoring advantage relative to the average of other PGA TOUR golfers (i.e., the field). We use a similar approach to rank PGA TOUR courses for their difficulty, both overall and in each part of the game. We also discuss the recent change in the groove rule for irons by the United States Golf Association. A preliminary analysis shows that it has had almost no impact on scores from the rough.

Key words: sports statistics; golf; PGA TOUR; Tiger Woods; dynamic programming; performance measurement; strokes gained.
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A golf score tells how well a golfer played overall; however, it does not reveal the factors that contributed most to that score. The goal of this paper is to analyze the play of PGA TOUR professional golfers to understand and quantify the contributions of three categories of golf shots—long game (shots over 100 yards from the hole), short game (shots under 100 yards from the hole, excluding putting), and putting—in determining a total golf score for an 18-hole round. We use this performance attribution analysis to rank golfers in various skill categories and also to examine the relative impact of each skill category on overall score.

Although golf fans know that Tiger Woods is the best golfer of his generation, they often debate whether his low scores are primarily because of superior putting, wedge play around the greens, driving, or some other factor or combination of factors. Sweeney (2008) writes: "What really differentiates Woods from everyone else is his ability to make more putts from the critical range of 10 to 25 feet." In June 2010, US Open winner Geoff Ogilvy said, "I think by now every player on tour is aware that the biggest reason Tiger is the best is because he putts the best" (Diaz 2010). In spite of these assertions, it is not clear that putting is the most important factor contributing to Tiger's scoring advantage.

This paper shows that Tiger Woods' scoring advantage in the years 2003–2010 was 3.20 strokes per round better than an average tournament field. He is ranked at or near the top of PGA TOUR golfers in the three categories; he dominates because he excels in all phases of the game. However, his long game accounts for 2.08 of the total 3.20 strokes gained per round; therefore, about two-thirds of his scoring advantage comes from shots over 100 yards from the green. His putting advantage versus the field is 0.70 shots per round, whereas his short game contributes 0.42 shots per round. Although he is a phenomenal putter, his gain from putting is less than the 1.01 strokes he gains from shots starting between 150 and 250 yards from the hole, and comparable to the 0.70 strokes he gains from long tee shots.

Performance attribution analysis is difficult using standard golf statistics, many of which involve relatively crude counting measures. For example, the fairways-hit statistic counts the number of fairways hit on a long tee shot (i.e., on par-4 and par-5 holes) divided by the number of tee shots. However, this statistic does not distinguish shots that barely miss the fairway from shots that miss by a large distance and land behind trees, in water, or out of bounds. In addition, many standard golf statistics combine several parts of the game. For example, the sand-save statistic counts the number of times a golfer gets the ball in the hole in one or two shots from a greenside sand bunker divided by the number of attempts. However, this statistic mixes sand play with putting, making it difficult to isolate these skills. Having shot location information is useful to better measure driving skill, sand play, and putting skill. The PGA TOUR has collected this type of detailed data in its ShotLink™ system since 2003.

In this paper, we use detailed shot data to assess and rank the performance of PGA TOUR golfers in the three categories of shots. The performance analysis is based on the concept of *strokes gained*, which measures the quality of each shot based on its starting and ending locations. Broadie (2008) discusses this concept using the term *shot value* in place of strokes gained. As Broadie and Ko (2009) point out, the strokes gained metric is related to the value function of a dynamic program. For example, if a golfer hits a poor sand shot followed by a great putt, the sand shot will have a negative strokes gained value, whereas the putt's strokes gained value will be positive. This approach allows each shot to be measured on its own merits; this is not possible using the sand-save statistic, which combines both shots. Just as golf scores are often compared to the benchmark of par, strokes gained represents the quality of a shot relative to a benchmark, as defined by the average performance of PGA TOUR golfers. Adding strokes gained for shots in a given category provides a performance measure for that category and is useful in understanding a golfer's strengths and weaknesses and in comparing golfers. Strokes gained analysis is used to determine the factors that separate the top golfers on the tour.

Since the publication of the landmark book by Cochran and Stobbs (1968), a large set of literature on the scientific and statistical analysis of golf has been developed. Recent surveys include Penner (2003), Farrally et al. (2003), and Hurley (2010). Riccio (1990) did a statistical analysis of amateur golfers. Landsberger (1994) made an early attempt to quantify the value of a shot. Several papers discuss the skill factors that are most important in determining earnings in professional tournaments. Examples include Davidson and Templin (1986), Shmanske (1992), Moy and Liaw (1998), Berry (1999, 2001), Nero (2001), Callan and Thomas (2007), Shmanske (2008), and Puterman and Wittman (2009). Most of these studies were limited by the lack of detailed shot information and had to rely on standard golf statistics (e.g., putting average, sand-save percentage, and fairways hit). The strokes gained approach, which directly decomposes a golfer's score by the quality of each shot, is an alternative to the regression analyses used in many earlier studies.

Broadie (2008) introduced strokes gained analysis primarily to determine the skills that separate the play of professional and amateur golfers. Fearing et al. (2010) also used strokes gained analysis to investigate putting performance on the PGA TOUR. In their study, they adjusted the putting benchmark to account for the distance to the hole, the difficulty of the green on each hole, and the quality of putters in each tournament. Larkey (1994) and Berry (2001) represent early efforts to adjust tournament results for course difficulty and golfer skill factors. More recently, Connolly and Rendleman (2008) used a statistical model to investigate golfer skill and streaky play on the PGA TOUR. The important concept in Larkey (1994), Berry (2001), Connolly and Rendleman (2008), and Fearing et al. (2010) is that overall scores and number of putts depend on golfer skill and course difficulty. Fewer putts are sunk on bumpy greens, and scores are higher on more difficult courses (e.g., those with narrow fairways, deep rough, and many water hazards). However, discerning the difficulty of a course is problematic when golfer skill is unknown. The issue of disentangling golfer skill from course difficulty in golf scores also arises in creating golf handicaps for amateur golfers. Pollock (1974), Scheid (1977), and Stroud and Riccio (1990) discuss issues related to golf handicapping.

This paper extends the analysis in Broadie (2008) in several ways. First, it estimates a benchmark function representing the average strokes to complete a hole for PGA TOUR golfers. This benchmark summarizes the skill of PGA TOUR golfers in various shot categories. A component of estimating the benchmark is the automatic identification of recovery shots. Second, it uses an estimation procedure to simultaneously estimate the difficulty of each course and round and to adjust the strokes gained results for the difficulty factors. In addition to providing a better measure of golfer performance, this procedure allows courses to be ranked by difficulty, both overall and in each part of the game. Finally, it applies the analysis to a database of more than eight million shots by PGA TOUR golfers, leading to interesting results, including the relative importance of the long game versus the short game.

Strokes Gained

Strokes gained is a simple and intuitive quantitative measure of the quality of a golf shot. Suppose we estimate a function, $J(d, c)$, where d represents the distance to the hole from the current location (not the distance of the shot), c represents the condition of the current ball location (i.e., green, tee, fairway, rough, sand, or recovery), and J is the average number of strokes a PGA TOUR golfer takes to finish the hole from the current location. For brevity, J is referred to as the benchmark. Define the *strokes gained* of the ith shot on a hole that starts at (d_i, c_i) and finishes at (d_{i+1}, c_{i+1}) to be

$$g_i = J(d_i, c_i) - J(d_{i+1}, c_{i+1}) - 1. \tag{1}$$

Strokes gained represents the decrease in the average number of strokes to finish the hole from the beginning of the shot to the end of the shot, minus one to account for the stroke taken. For example, suppose the average number of shots to complete the hole is 2.6 from a position in the fairway 40 yards from the hole. If a golfer hits the shot to one foot from the hole, where the average number of shots to complete the hole is 1.0, then Equation (1) attributes a gain of 0.6 strokes to the shot: it reduces the average number of shots to complete the hole by 1.6 and took one shot to do so, for a gain of 0.6. In general, a positive g_i indicates that a shot is better than a PGA TOUR golfer's average shot, whereas a negative g_i indicates that a shot is worse than average.

The units of strokes gained are golf strokes (e.g., a strokes gained value of -0.1 means the shot is 0.1 strokes worse than the benchmark). Because the units are the same for all shot types, the strokes gained metric offers a consistent method for evaluating different aspects of the game. It solves the problem of incommensurable measures in standard golf statistics, as Larkey and Smith (1999) point out.

For example, suppose that PGA TOUR golfer A plays a long par-3 that takes the PGA TOUR field an average of 3.2 strokes to complete the hole. Golfer A's tee shot finishes on the green, leaving a 16-foot putt for birdie. From 16 feet, the PGA TOUR field takes an average of 1.8 putts to finish the hole. The PGA TOUR field will one-putt about 20 percent of the time, two-putt about 80 percent of the time, and, rarely, three-putt from 16 feet ($1.8 = 20\%(1) + 80\%(2) + 0\%(3)$). The ball started in a spot at which the benchmark is 3.2 and finished at a position at which the benchmark is 1.8; therefore, the strokes gained for the shot is ($3.2 - 1.8 - 1 = +0.4$). Golfer A left his birdie putt one inch short. His ball started in a spot at which the benchmark is 1.8 and finished in a spot at which the benchmark is 1 (the average number of shots to finish the hole for a tap-in is 1), for a strokes gained value of ($1.8 - 1 - 1 = -0.2$). Golfer A's missed putt represents a loss of 0.2 shots relative to the benchmark, because he reduced the average number of strokes to complete the hole by 0.8, but he used one putt to do so. Because a PGA TOUR golfer sinks only 20 percent of 16-footers, missing this putt does not cost a full shot: it only costs 0.2 strokes. To complete the example, golfer A tapped in for par. The strokes gained equation, Equation (1), gives a value of zero for this putt, because he reduced the benchmark from one to zero using one shot. This makes sense, because sinking a one-inch putt neither gains nor loses shots relative to the benchmark.

The strokes gained metric has a simple but important *additivity property:* the strokes gained of a group of shots is the sum of the strokes gained of the individual shots. Suppose a golfer takes n shots on a hole. The total strokes gained for the n shots is $\sum_{i=1}^{n} g_i = \sum_{i=1}^{n}(J(d_i, c_i) - J(d_{i+1}, c_{i+1}) - 1) = J(d_1, c_1) - n$, because of the telescoping sum, and $J(d_{n+1}, c_{n+1}) = 0$ for the last shot, which ends in the hole. In the previous example, golfer A's score of $n = 3$ represents a net gain of 0.2 strokes compared to the benchmark of $J(c_1, d_1) = 3.2$ from the tee. Golfer A did this with a great tee shot (+0.4 strokes gained), a disappointing putt (−0.2 strokes gained), and a tap-in (0 strokes gained), for a total strokes gained of 0.2 for the hole, consistent with the additivity property.

Let us consider PGA TOUR golfer B playing the same par-3 hole. Golfer B's tee shot missed the green long and left. From this position in the rough, suppose the average number of shots to complete the hole (the benchmark) is 2.6. The strokes gained equation, Equation (1), gives $(3.2 - 2.6 - 1 = -0.4)$; therefore, golfer B lost 0.4 strokes compared to the PGA benchmark. Golfer B hit his second shot from the rough to inside of four feet from the hole, where the benchmark score is 1.1 (a PGA TOUR golfer sinks about 90 percent of these putts). Applying Equation (1) gives $(2.6 - 1.1 - 1 = 0.5)$; therefore, golfer B's second shot gained a half-stroke compared to the benchmark. Golfer B sunk the four-footer, and the strokes gained equation gives $(1.1 - 0 - 1 = 0.1)$. Golfer B's score of 3 also represents a net gain of 0.2 strokes compared to the benchmark value of 3.2 from the tee. Golfer B did this with a poor tee shot (−0.4 strokes gained), a good chip from the rough (0.5 strokes gained), and a one-putt (0.1 strokes gained), for a total of 0.2 strokes gained for the hole.

Golfers A and B scored the same on the hole, but did it in very different ways. If this was a representative example, we could see that golfer A has a great long game, while golfer B has a great short game. Strokes gained allows us to compare golfer A's game to golfer B's, both in total strokes gained (for the hole, round, or season) and in various categories (e.g., long game, short game, and putting). In a similar way, we can decompose the strokes gained for a given category into subcategories. For example, the total strokes gained of all long-game shots can be split into the sum of strokes gained for tee shots and approach shots from various distance categories. Unlike fraction of greens hit, proximity to the hole, or other statistical measures, the strokes gained approach provides a consistent way to quantify the value of shots in various categories and subcategories.

The game of golf can be modeled as a dynamic program. The score on a hole depends on the strategy and results of each shot on the hole. The optimal strategy from the tee depends on all possible outcomes of the first shot and the optimal strategy for the second shot, which depend on all of the possible outcomes of the second shot and the optimal strategy for the third shot, etc. The solution of a dynamic program involves starting from the last stage in this case, the shot that ends in the hole, and working backward to determine the optimal strategy. The Bellman (1957) equation says

$$J(d_i, c_i) = \min_{\mu} \mathrm{E}\big[J(d_{i+1}, c_{i+1}) + 1 \mid (d_i, c_i, \mu)\big], \qquad (2)$$

where the expectation is taken over (d_{i+1}, c_{i+1}), the random distance and condition of the end of shot i, given its start at (d_i, c_i), and the strategy μ (e.g., target and club) chosen by the golfer. For more detail, see Broadie and Ko (2009). This paper does not address the strategy choices of golfers; however, because PGA TOUR golfers are among the best golfers in the world, we can reasonably assume that they play optimal or nearly optimal strategies, and we can use the observed data to estimate $(J(d_i, c_i) = \mathrm{E}[J(d_{i+1}, c_{i+1}) + 1 \mid (d_i, c_i, \mu^*)])$, where μ^* represents an optimal strategy. The quality of an individual shot can be measured by the difference in the left and right sides of the equation for a particular outcome (i.e., by $J(d_i, c_i) - J(d_{i+1}, c_{i+1}) - 1$), which is the strokes gained definition given in Equation (1). This dynamic program viewpoint provides the justification for the strokes gained definition.

PGA TOUR Benchmark

The strokes gained computation is based on a benchmark function that gives the average number of strokes for a PGA TOUR golfer to complete a hole. The benchmark typically increases with the distance to the hole and depends on the course condition at the location of the ball (i.e., tee, fairway, rough, green, sand, or recovery). Because shots from the rough are more difficult than shots from the fairway, the benchmark is larger. In some situations, typically from the rough, a direct shot to the hole is impossible because the path is blocked by trees or other obstacles. In such cases, a golfer may elect to play a recovery shot—a short shot that is hit back to the fairway rather than directly toward the hole. Recovery shots are placed in their own category to better estimate the differential effects of fairway and rough. This section discusses the estimation of the benchmark function, the recovery shot identification procedure, and empirical results.

The results in this paper are based on the PGA TOUR's extensive ShotLink database, which includes all shots at PGA TOUR tournaments from 2003 to 2010. The database contains more than eight million shots (about one million shots per year), with shot locations measured to within one inch on putts and one foot on other shots. Deason (2006) gives additional information on the ShotLink system. The database does not include detailed shot information for the four major tournaments: the Masters, US Open, British Open, and the PGA.

The benchmark function (the average number of shots to complete the hole) must be defined in terms of observable information recorded in the database. Not all shots from the fairway with 125 yards to the hole are equal in difficulty. Many other factors are involved; for example, the ball's lie might be perfect or in a divot, the golfer's stance might be level or on a hill, or the wind could be calm or gusting, all of which affect the difficulty of a shot and the average number of shots to complete the hole. However, the benchmark can only be computed from observable information, and the ShotLink database includes the most important of these factors: the distance from the hole and the condition of the ball (e.g., tee, fairway, green, sand, or rough). The benchmark function can be interpreted as an average over these other unobservable factors.

The goal is to estimate the benchmark function ($J(d, c)$), where d represents the distance to the hole from the current location and c represents the condition of the current location (i.e., green, tee, fairway, rough, sand, or recovery). Statistical and model-based approaches are the two main ways to accomplish this. Statistical procedures include simple interpolation, linear regression, splines, kernel smoothing, and other methods. In model-based approaches, a parametric analytical or simulation model is formulated and optimization is used to determine the model parameters that best fit the data. Both approaches attempt to find a benchmark that is close to the data and appropriately smooth to take into account the noise in the data.

The database's large size allows for accurate estimation of the benchmark because in most distance and condition categories, many shots are available to estimate the average score to complete the hole. Experimenting with several approaches yielded similar results. Piecewise-polynomial functions were used as the form of the benchmark, except for putts on the green; for these, a model-based approach was used to fit one-putt probabilities based on a simplification of the putting model presented in Broadie and Bansal (2008). This was combined with a statistical model for three-putts to give an average score function for putts on the green. This approach can be used to smooth the somewhat limited data for long putt distances.

Tee Shot Benchmark

From the tee, a simple linear regression of average score (J) on the distance to the hole (d, measured in yards) for PGA TOUR professionals (pros) using 2003–2010 data gives ($J = 2.38 + 0.0041d$). The distance to the hole d is measured along the fairway from the tee to the hole (i.e., the dogleg, not the direct distance). In this regression, the data are grouped into 20-yard distance buckets, and the R^2 of the regression is over 98 percent. The slope of the equation implies that each additional 100 yards of hole distance adds 0.41 strokes to the average score of a PGA TOUR pro. This regression is similar to the result ($2.35 + 0.0044d$) obtained in Cochran and Stobbs (1968), based on a smaller set of data collected from a single British professional tournament in 1964.

In spite of the high R^2, a linear regression does not provide an adequate fit to the data, as Figure 1 shows. In particular, the average score from the tee exhibits a jump between long par-3 holes at 235 yards and short par-4 holes at 300 yards (little data exist between these distances). The computations in the paper are based on a more accurate piecewise polynomial fit to the data (see Table B.1 in Appendix B).

Broadie (2008) finds that the average score from the tee is ($2.79 + 0.0066d$) for golfers whose 18-hole average score is 90 (i.e., 90-golfer). The slope implies that each additional 100 yards of hole distance adds 0.66 strokes to the average score of 90-golfers; for PGA TOUR pros, it adds 0.41 strokes. The United States Golf Association (USGA) refers to this slope as the *ability to overcome distance*. For 90-golfers, going from 180 yards (par-3 distance) to 580 yards (par-5 distance) increases their average score by about 2.6. However, the par increases by 2; therefore, 90-golfers do *worse* relative to par on par-5 holes compared to par-3 holes. Pros who go from a hole of 180 yards

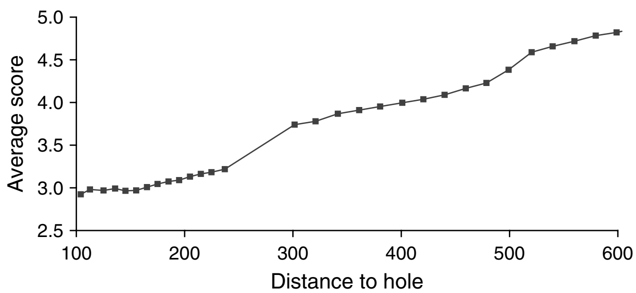

Figure 1: The graph shows the average score from the tee for PGA TOUR golfers in 2003–2010. Distance to the hole is measured along the fairway from the tee to the hole, not directly.

(par-3 distance) to 580 yards (par-5 distance) will see an average score increase of 1.6. The par goes up by 2; therefore, the pros do *better* relative to par on par-5 holes compared to par-3 holes. The main reason is the 290-yard average distance that the pros drive the ball, compared to an average drive of about 210 yards for 90-golfers. (Of course, on par-5 holes, 90-golfers have more chances to flub shots or hit into trouble.)

Benchmark Within 50 Yards of the Hole

In this subsection, we compare average strokes to complete the hole from the sand, rough, and fairway on shots within 50 yards of the hole. We often hear that professional golfers are so good from the sand that they would rather be in the sand than in the rough. Figure 2 illustrates the data and the fitted curves. The figure shows that when the distance to the hole is less than 15 yards or greater than 34 yards, sand shots have *larger* average strokes to complete the hole than shots from the rough from the same distance. In the range from 15 yards to 34 yards, sand shots are easier than shots from the rough, on average. Conditioned on the shot starting within 50 yards of the hole, the average initial distance to the hole for shots from the sand and rough is 16 yards, just about the distance of equal difficulty for sand and rough shots.

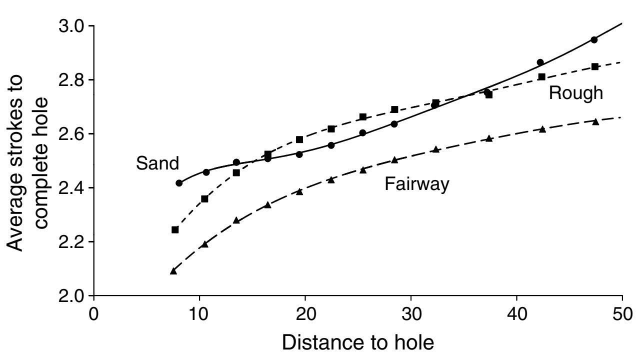

Figure 2: The graph shows the average strokes to complete the hole from the rough, sand, and fairway for PGA TOUR golfers in 2003–2010.

The average score can be translated into an up-and-down fraction, that is, the fraction of the time it takes a golfer to finish a hole using two or fewer shots. From 15 yards from the hole, pros get up and down 51 percent of the time from the rough or sand and 69 percent of the time from the fairway. At 25 yards from the hole, pros get up and down 42 percent of the time from the sand, 35 percent from the rough, and 54 percent from the fairway. These are averages over all situations; note that the outcome for an individual shot will depend on the ball's lie, the contour of the green near the hole, and other factors. However, the distance from the hole and condition of the ball are primary factors in determining the average number of shots to complete the hole.

Putting Benchmark

In this subsection, the estimation of the benchmark function for putts is discussed. The benchmark is fit in three steps. First, a one-putt probability function is fit, then a three-putt function is fit, and then these two are combined into a benchmark average putts-to-complete-the-hole function. This approach is followed for several reasons. First, the data are sparse and noisy for long putts (e.g., greater than 50 feet from the hole), and so smoothing is necessary. Second, the procedure works well for fitting smaller data sets and it is useful to have a consistent procedure for all sets of data. Finally, golfers think in terms of one-putts and three-putts, so these models and results are of independent interest. Details are given in Appendix A. Based on this putting benchmark and strokes gained methodology, the PGA TOUR introduced strokes gained-putting in May 2011.

The left panel of Figure 3 illustrates the data and the fitted one-putt probability curve. The model probability is almost always within one standard error of the data. (The standard errors are too small to show clearly on the graph.) PGA TOUR golfers one-putt

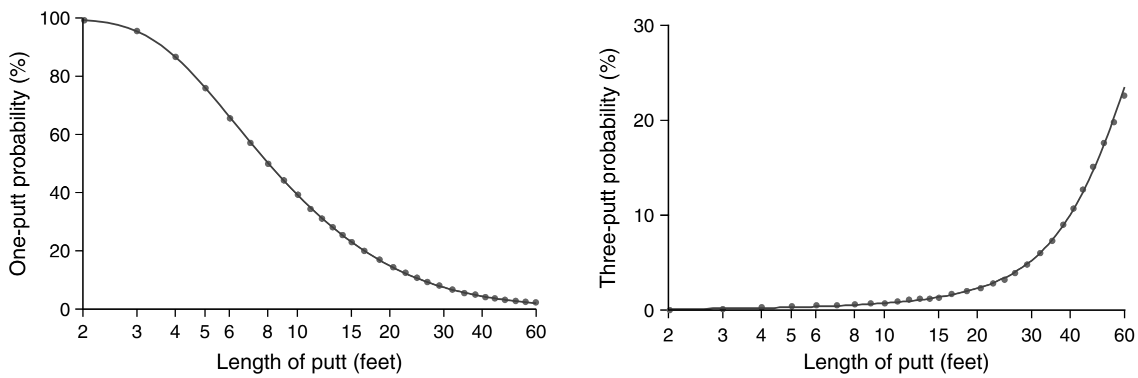

Figure 3: PGA TOUR putting results using 2003–2010 data are shown in the graphs. Left panel: one-putt probability. Right panel: three-putt probability. Dots represent the data, and the curves are the fitted models.

50 percent of the time from a distance of eight feet. Comparing this result from 2009 data with earlier results is interesting. In a fairly small sample, Cochran and Stobbs (1968, Chapter 29) found that pros one-putted 50 percent of the time from a distance of seven feet in 1964. Using data from the early 1960s on regular tournament courses, Soley (1977, Chapter 4) found that pros sunk 50 percent of their putts from seven feet. He found the same result at the 1974 US Open at Winged Foot, but found that the distance was closer to six feet at the 1972 US Open at Pebble Beach. Using hand-collected data from PGA tournaments in the 1980s, Pelz found that pros sunk 50 percent of their putts from about six or seven feet (Pelz 1989 p. 38; 2000, p. 7). The increase in the 50 percent one-putt distance from six or seven feet to the current eight feet could be because of better-conditioned greens, better putting skill, or a combination of both factors. By contrast, amateur golfers with an average 18-hole score of 90 (90-golfers) one-putt 50 percent of the time from five feet.

The right panel of Figure 3 illustrates the data and the fitted three-putt probability curve. The three-putt probability for PGA TOUR golfers does not exceed 10 percent until 40 feet. PGA TOUR golfers average 0.55 three-putt greens per round—about 2.2 per four-round tournament. Amateur 90-golfers three-putt about 2.3 times per round—four times more often than pros. Figure 4 shows how the average number of putts increases with distance for PGA TOUR golfers. They average two putts from 33 feet (i.e., the fraction of one-putts equals the fraction of three-putts). Amateur 90-golfers average two putts from 19 feet.

Recovery Shots

A shot is called a recovery shot if the golfer's shot to the hole is impeded by trees or other obstacles. Even if the golfer decides to hit toward the hole through a small opening in trees, or attempts a hook or slice around an obstacle, it is still considered a recovery shot because the golfer is recovering from trouble. In this subsection, we discuss recovery shots, their importance in the benchmark, and their identification.

Suppose a golfer hits a long drive that ends up behind a tree and is forced to chip back out onto the fairway for the second shot (i.e., the second shot is a recovery shot). If the benchmark does not account for recovery shots, the strokes gained of the tee shot may be close to zero, but the second shot will have

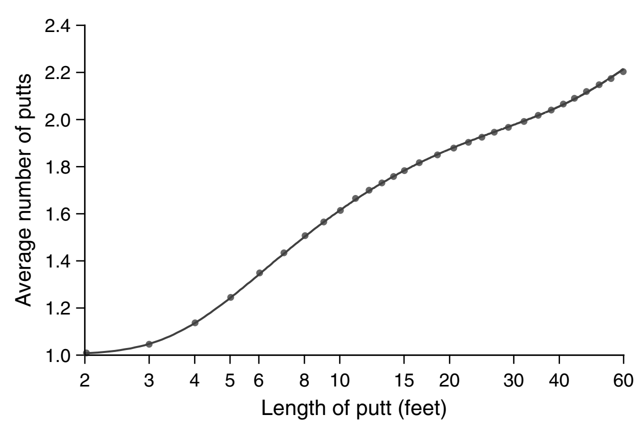

Figure 4: The graph shows the average number of putts by initial distance to the hole for PGA TOUR golfers in 2003–2010. Dots represent the data, and the curve is the fitted benchmark model.

a negative strokes gained because it did not travel very far. This makes little sense; the problem was caused by a poor tee shot, not a poor second shot. The strokes gained equation can account for this situation by identifying the condition of the second shot as a recovery shot, and the benchmark will have a larger average number of strokes to complete the hole than from a comparable distance in the rough. Using a separate benchmark for recovery shots will give a negative strokes gained for the poor tee shot and a strokes gained of close to zero for the second shot. The recovery label is important for correctly allocating strokes gained between the two shots and for estimating the penalty for being in the rough versus the fairway. The rough was not the direct cause of the increase in score; it was an obstructed route to the hole.

The ShotLink database does not have an identifier for a recovery shot. Labeling a shot as a recovery is a judgement call, unlike distance to the hole, which is an observable and objective quantity. The recovery-shot condition must be inferred from existing information in the data. However, because the database contains millions of shots, a manual identification procedure is infeasible. The automatic recovery-shot identification procedure has two steps. The first finds shots that travel an unusually short distance (e.g., less $r_1 = 40$ percent of the distance to the hole) or are hit at a large angle relative to the hole (e.g., an absolute angle greater than $r_2 = 15$ degrees with respect to the ball hole line). Shots are also screened to start at a minimum of $r_3 = 30$ yards from the hole. The parameters are determined by visually inspecting a number of shots that satisfy the criteria. The second step finds shots that start close (e.g., within $r_4 = 3$ yards) to shots by other golfers, which are labeled as recovery shots in Step 1. For example, suppose two golfers are in nearly identical recovery-shot positions obstructed by trees. The first golfer chips back onto the fairway, and the second golfer attempts a big slice around the trees. Step 1 would identify the first golfer's shot as a recovery shot; however, this step might not identify the second golfer's shot as a recovery, although it started in the same position and was significantly affected by trees. The second step of the procedure allows the second golfer's shot to be labeled as recovery.

Figure 5 illustrates three shots labeled as recovery shots by this procedure. Golf course images from Google Earth are used to display the shots. The ShotLink database contains shot starting and ending positions using (x, y) coordinates, which we translated to latitude and longitude for plotting. Although this method of inferring which shots are recovery shots works well, two types of errors can occur. Some nonrecovery shots will be labeled as recovery; some recovery shots will not be labeled as such. Given the current data and judgement involved, designing an error-proof procedure is impossible. However, the magnitudes of the two types of errors can be controlled by the parameter choices. Recovery-shot identification is important when comparing the average number of shots to complete a hole from the rough versus the fairway. Once recovery shots are identified, benchmark functions are fit to the data using piecewise polynomials (see Table B.1 in Appendix B). Figure 6 shows the average strokes to complete the hole for recovery shots and shots from the rough and fairway.

PGA TOUR Golfer Rankings and Results

In this section, we use strokes gained analysis to rank PGA TOUR golfers in various skill categories and subcategories. The strokes gained are first adjusted by the course difficulty for that round to produce more reliable comparisons between golfers. This section provides details of the adjustment procedure, results, and discussion of the rankings, and analyzes which skill factors determine the best golfers on the tour.

Course-Round Difficulty Adjustments

Some four-round PGA tournaments have winning scores of 30 under par; winning scores for others might be only 6 under par. The difference of six shots per round is because of two main factors: course difficulty and weather conditions. The difference in difficulty can be due to length of the course, width of the fairways, firmness of the greens, height of the rough, severity of bunkers, and other factors. Weather conditions, especially wind, can dramatically affect a ball's flight. However, the benchmark average score directly includes only the course length; therefore, a golfer who shoots 12 under par in a tournament with a winning score of 30 under par is likely to have

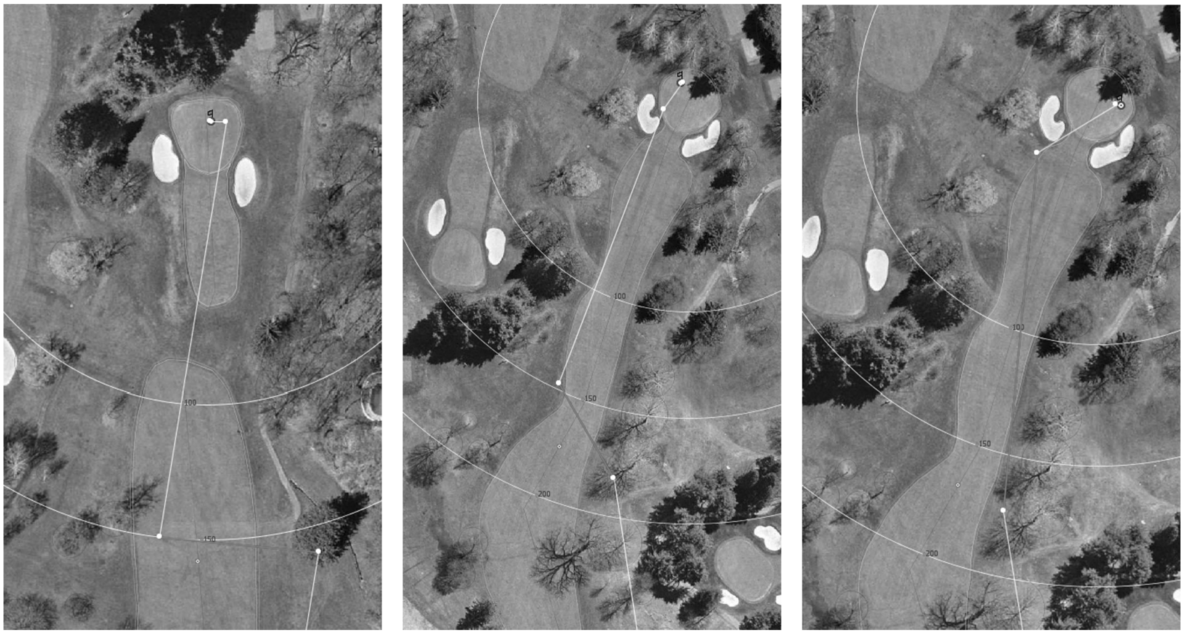

Figure 5: The three photographs illustrate recovery shots. Left panel: Corey Pavin, 6/11/2006, hole 3, Westchester Country Club. The shot indicated is labeled a recovery shot because of the distance criterion. Middle panel: Tim Clark, 6/26/2005, hole 15, Westchester Country Club. The shot indicated is labeled a recovery shot because of the distance criterion. Right panel: Fred Couples, 6/11/2006, hole 15, Westchester Country Club. The shot indicated would not be labeled a recovery shot by the distance or angle criterion, but it is labeled a recovery shot because it is near another golfer's recovery shot (not shown). Arcs show 100-, 150-, and 200-yard distances from the hole. *Source*: Google Earth.

played relatively worse than a golfer with a score of 4 under par when the winning score is 6 under par. To make a direct comparison of two golfers who play in a different set of tournaments, adjusting scores, and

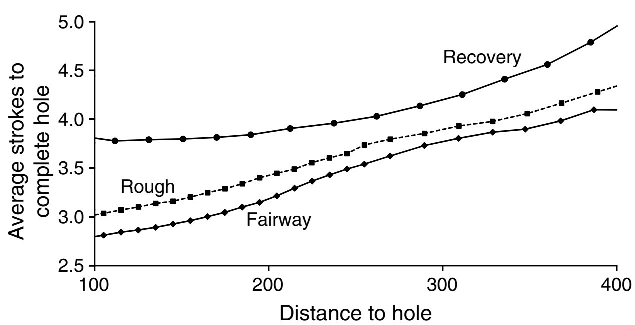

Figure 6: The graph shows the average strokes to complete the hole for recovery shots and shots from the rough and fairway for PGA TOUR golfers in 2003–2010. Most recovery shots are in the range between 150 and 300 yards from the hole. In this range, the average number of strokes to complete the hole is 0.6 strokes greater from a recovery position than from the fairway and 0.4 greater than from the rough.

strokes gained for the course difficulty for each round is necessary.

Let g_{ij} represent the total (18-hole) strokes gained for golfer i playing on a course and round indexed by j. To separate golfer skill from course difficulty for that round, the strokes gained, g_{ij}, is modeled as

$$g_{ij} = \mu_i + \delta_j + \epsilon_{ij}, \tag{3}$$

where μ_i represents golfer i's intrinsic skill (i.e., the golfer's average strokes gained on a PGA TOUR course of average difficulty), δ_j represents the intrinsic difficulty of the course-round j, and ϵ_{ij} is a random mean zero-error term. The model is estimated using a standard iterative procedure, as Larkey (1994), Berry (2001), and Connolly and Rendleman (2008) discuss.

Golfer Strokes Gained Results and Rankings

Tables 1–3 show the main golfer results. Table 1 shows PGA TOUR golfer rankings based on the 2003–2010 data. Ranks are relative to the 299 golfers with 120 or more rounds in the data. (The rankings are based on strokes gained per round. An argument can be made that a better measure of skill is strokes gained per stroke; however, both approaches give similar results. We use strokes gained per round because the additivity property makes it easier to see how total strokes gained splits into long-game, short-game, and putting strokes gained.) Tiger Woods' total strokes gained per round is 3.20, which means that he gains, on average, 3.20 strokes per 18-hole round versus an average PGA TOUR field. That is, 3.20 represents the μ for Tiger Woods, as estimated from Equation (3). Tiger is ranked first in this category; Jim Furyk, who gains 2.12 strokes per 18-hole round versus an average PGA TOUR field, occupies second place. The difference between these two golfers is an enormous 1.08 strokes per round. Differences between lower ranks are much smaller: the average difference is 0.08 strokes between ranks 2 and 10 and 0.01 strokes between ranks 95 and 105. Between 2003 and 2010, Tiger is the best golfer by a large margin.

The strokes gained approach gives direct insight into where Tiger Woods gained the 3.20 strokes per round. Table 1 shows that 2.08 strokes came from the long game (rank 1), 0.42 strokes from the short game (rank 16), and 0.70 strokes from putting (rank 3). Tiger dominates the competition because he excels in every category, but his long game contributes 65 percent (2.08/3.20) to his total strokes gained relative to an average field. Many people have commented on

	Rank				Strokes gained			
Golfer	Total	Long	Short	Putt	Total	Long	Short	Putt
Woods, Tiger	1	1	16	3	3.20	2.08	0.42	0.70
Furyk, Jim	2	10	10	14	2.12	1.13	0.47	0.52
Singh, Vijay	3	2	5	195	2.05	1.63	0.51	−0.09
Els, Ernie	4	4	15	153	1.86	1.40	0.44	0.01
Mickelson, Phil	5	12	12	95	1.72	1.11	0.47	0.15
Donald, Luke	6	65	7	9	1.55	0.46	0.50	0.58
Goosen, Retief	7	19	22	46	1.52	0.90	0.33	0.29
Garcia, Sergio	8	5	60	220	1.47	1.39	0.23	−0.15
Scott, Adam	9	7	53	201	1.46	1.33	0.24	−0.11
Harrington, Padraig	10	54	4	42	1.44	0.57	0.56	0.31
Average					1.84	1.20	0.42	0.22
Boros, Guy	290	283	292	91	−1.14	−0.87	−0.43	0.16
McGovern, Jim	291	293	158	197	−1.15	−1.05	−0.01	−0.09
Waite, Grant	292	279	120	282	−1.17	−0.79	0.07	−0.45
Begay III, Notah	293	265	194	286	−1.23	−0.67	−0.09	−0.48
Bolli, Justin	294	267	274	264	−1.27	−0.69	−0.25	−0.32
Veazey, Vance	295	294	246	178	−1.33	−1.10	−0.19	−0.04
McCallister, Blaine	296	262	273	294	−1.49	−0.64	−0.25	−0.60
Gossett, David	297	292	103	295	−1.49	−1.01	0.12	−0.61
Duval, David	298	297	219	143	−1.51	−1.41	−0.14	0.03
Perks, Craig	299	298	195	249	−1.79	−1.44	−0.09	−0.26
Average					−1.36	−0.97	−0.12	−0.27
Notable golfers								
Couples, Fred	29	28	37	209	1.00	0.84	0.28	−0.12
Villegas, Camilo	30	13	126	212	0.99	1.05	0.06	−0.13
Westwood, Lee	43	17	253	129	0.83	0.97	−0.20	0.06
Pavin, Corey	99	252	8	26	0.33	−0.57	0.48	0.42
Durant, Joe	117	9	267	299	0.20	1.14	−0.24	−0.70
O'Meara, Mark	236	284	89	62	−0.49	−0.87	0.15	0.24
Daly, John	238	138	254	272	−0.50	0.08	−0.21	−0.38

Table 1: The table reports total strokes gained per round, broken down into three categories: long game, short game, and putting. Ranks are based on the 299 PGA TOUR golfers with at least 120 rounds during 2003–2010.

Golfer	Rank: Long total	Rank: Long tee	Rank: 100–150	Rank: 150–200	Rank: 200–250	Rank: > 250	Strokes gained per round: Long total	Strokes gained per round: Long tee	Strokes gained per round: 100–150	Strokes gained per round: 150–200	Strokes gained per round: 200–250	Strokes gained per round: > 250
Woods, Tiger	1	7	8	1	1	1	2.08	0.70	0.20	0.66	0.35	0.14
Singh, Vijay	2	3	20	8	7	14	1.63	0.81	0.16	0.33	0.19	0.07
Allenby, Robert	3	14	4	6	2	47	1.59	0.61	0.25	0.38	0.26	0.05
Els, Ernie	4	16	14	2	15	26	1.40	0.55	0.18	0.41	0.16	0.06
Garcia, Sergio	5	15	13	13	4	17	1.39	0.55	0.18	0.31	0.23	0.07
Perry, Kenny	6	6	25	10	18	95	1.37	0.73	0.15	0.32	0.15	0.03
Scott, Adam	7	18	5	12	48	16	1.33	0.54	0.25	0.31	0.10	0.07
Weekley, Boo	8	2	58	84	25	113	1.19	0.83	0.09	0.09	0.13	0.02
Durant, Joe	9	11	43	22	40	136	1.14	0.67	0.11	0.24	0.11	0.01
Furyk, Jim	10	51	7	4	12	134	1.13	0.32	0.21	0.40	0.18	0.01
Average							1.40	0.61	0.17	0.35	0.19	0.06
Notable golfers												
Couples, Fred	28	28	22	92	26	117	0.84	0.47	0.16	0.08	0.12	0.02
Daly, John	138	55	242	252	123	128	0.08	0.31	−0.08	−0.15	0.03	0.02
Faxon, Brad	289	297	79	190	223	279	−0.96	−0.82	0.07	−0.05	−0.05	−0.08
Duval, David	297	298	249	211	293	164	−1.41	−1.08	−0.09	−0.08	−0.17	−0.00

Table 2: The table reports long-game strokes gained per round, broken down into five categories: long tee shots (tee shots starting over 250 yards from the hole), approach shots 100–150 yards from the hole, approach shots 150–200 yards from the hole, approach shots 200–250 yards from the hole, and shots over 250 yards from the hole (excluding tee shots). To conserve space, recovery shots and sand shots greater than 100 yards from the hole are not reported (but are included in the total long-game strokes gained). Ranks are based on the 299 golfers with at least 120 rounds during 2003–2010.

Golfer	Rank: Short	Rank: 0–20	Rank: 20–60	Rank: 60–100	Rank: Sand	Strokes gained per round: Short	Strokes gained per round: 0–20	Strokes gained per round: 20–60	Strokes gained per round: 60–100	Strokes gained per round: Sand
Stricker, Steve	1	7	1	1	59	0.69	0.19	0.22	0.17	0.08
Olazabal, Jose Maria	2	1	27	66	7	0.57	0.30	0.10	0.04	0.15
Riley, Chris	3	9	4	45	3	0.56	0.18	0.15	0.06	0.15
Harrington, Padraig	4	5	11	4	39	0.56	0.21	0.12	0.15	0.10
Singh, Vijay	5	14	9	53	4	0.51	0.15	0.12	0.05	0.15
Weir, Mike	6	53	15	40	1	0.51	0.09	0.11	0.06	0.21
Donald, Luke	7	3	84	49	2	0.50	0.23	0.04	0.06	0.17
Pavin, Corey	8	4	30	20	23	0.48	0.21	0.09	0.08	0.12
Imada, Ryuji	9	18	21	38	19	0.48	0.15	0.10	0.06	0.12
Furyk, Jim	10	6	17	13	66	0.47	0.20	0.11	0.09	0.07
Average						0.53	0.19	0.12	0.08	0.13
Notable golfers										
Haas, Jay	11	11	143	10	8	0.47	0.17	0.01	0.11	0.14
Mickelson, Phil	12	15	6	42	20	0.47	0.15	0.13	0.06	0.12
Woods, Tiger	16	22	8	47	64	0.42	0.13	0.13	0.06	0.07
Garcia, Sergio	60	64	75	60	101	0.23	0.08	0.05	0.04	0.04
Westwood, Lee	253	260	162	88	286	−0.20	−0.10	−0.00	0.02	−0.14
Daly, John	254	290	182	179	90	−0.21	−0.17	−0.01	−0.01	0.05

Table 3: The table gives short-game strokes gained per round, broken down into three distance categories: 0–20 yards from the hole, 20–60 yards from the hole, and 60–100 yards from the hole (excluding sand and recovery shots and putts). Greenside sand shots within 50 yards of the hole ("sand") are given in a separate category. To conserve space, 0–100 yard recovery shots and 50–100 yard sand shots are not reported (but are included in the total short-game strokes gained). Ranks are based on the 299 golfers with at least 120 rounds during 2003–2010.

his superior putting; the strokes gained analysis is consistent with this observation: he is ranked third with a gain of 0.70 putts per round. However, his gain from putting is less than the 1.01 strokes he gains between 150 and 250 yards from the hole, and comparable to his long tee shots, where he gains 0.70 strokes per round versus the field.

Table 1 shows average strokes gained for the top 10 golfers, and the long game contributes 65 percent (1.20/1.84) to their total strokes gained relative to an average field. The bottom 10 golfers, ranks 290–299, lose 71 percent (−0.97/−1.36) of their strokes in the long game. The top 10 golfers in total strokes gained are all ranked in the top 70 in long-game strokes gained; however, four of these golfers are not ranked in the top 100 in putting. The bottom 10 golfers in total strokes gained are all ranked worse than 200 in long-game strokes gained. These results suggest that the long game is the most important factor differentiating PGA TOUR golfers.

Table 2 focuses on the long game, and shows that Tiger is ranked in the top 10 in each long-game subcategory. Of his 2.08 long-game strokes gained, 1.01 strokes are gained between 150 and 250 yards from the hole. Although he is known for his occasional wild drives, in the long tee shot category he is ranked 7 (of 299), and he gains 0.70 strokes per round versus the field because he gains from putting.

Table 3 shows short-game strokes gained results during 2003–2010. Steve Stricker had the best short game overall, whereas Mike Weir and Luke Donald had the best greenside sand games. Table 4 focuses on putting and shows that David Frost, Brad Faxon, and Tiger Woods were the top three putters. Sergio Garcia is ranked 220 in putting overall: 271 in short putts, 179 in medium putts, and 85 in long putts. Clearly, the shorter the putt, the more trouble he has. Sergio's total putting strokes gained is −0.15; thus, he loses 0.85 strokes per round to Tiger Woods from putting only. Brad Faxon gains 0.71 strokes on the field in putting, but loses 0.96 in the long game (see Table 2).

Table 5 shows strokes gained results for Tiger Woods by year. He was ranked first in total strokes gained in each year from 2003 to 2009. However, he had the worst year of his career in 2010, with his total strokes gained per round decreasing by three compared with 2009. His game faltered across the board, dropping 1.19 strokes in his long game,

	Rank				Strokes gained per round			
Golfer	Putt	0–6 ft	7–21 ft	22+ ft	Putt	0–6 ft	7–21 ft	22+ ft
Frost, David	1	83	1	1	0.72	0.08	0.42	0.22
Faxon, Brad	2	21	3	2	0.71	0.19	0.31	0.21
Woods, Tiger	3	11	4	3	0.70	0.21	0.31	0.19
Crane, Ben	4	1	10	24	0.67	0.29	0.27	0.11
Roberts, Loren	5	4	13	13	0.65	0.25	0.26	0.14
Baddeley, Aaron	6	9	9	7	0.64	0.22	0.27	0.15
Chalmers, Greg	7	2	14	37	0.62	0.27	0.26	0.09
Parnevik, Jesper	8	3	27	9	0.61	0.25	0.21	0.15
Donald, Luke	9	14	17	16	0.58	0.20	0.24	0.13
Cink, Stewart	10	28	7	22	0.58	0.17	0.29	0.12
Average					0.65	0.21	0.28	0.15
Notable golfers								
Stricker, Steve	19	15	60	19	0.46	0.20	0.13	0.13
Pavin, Corey	26	97	23	17	0.42	0.06	0.22	0.13
Mickelson, Phil	95	68	139	102	0.15	0.09	0.02	0.04
Singh, Vijay	195	152	252	97	−0.09	0.01	−0.14	0.04
Couples, Fred	209	294	102	41	−0.12	−0.28	0.07	0.08
Garcia, Sergio	220	271	179	85	−0.15	−0.16	−0.03	0.04
Daly, John	272	261	272	247	−0.38	−0.12	−0.19	−0.07

Table 4: The table gives putting strokes gained per round, broken down into three distance categories: short putts (0–6 feet), medium-length putts (7–21 feet), and long putts (22 feet and over). Ranks are based on the 299 golfers with at least 120 rounds during 2003–2010.

Year	Rank Total	Rank Long	Rank Short	Rank Putt	Strokes gained Total	Strokes gained Long	Strokes gained Short	Strokes gained Putt
Tiger Woods								
2010	48	28	160	91	0.71	0.83	−0.20	0.08
2009	1	1	4	2	3.70	2.02	0.70	0.99
2008	1	1	3	4	4.14	2.56	0.72	0.85
2007	1	1	24	2	3.68	2.47	0.41	0.80
2006	1	1	16	21	3.78	2.83	0.45	0.49
2005	1	1	98	5	2.82	2.03	0.09	0.70
2004	1	5	11	3	3.07	1.62	0.49	0.96
2003	1	2	3	16	3.71	2.44	0.72	0.55
2003–2010	1	1	16	3	3.20	2.08	0.42	0.70

Year	Rank Long total	Rank Long tee	Rank 100–150	Rank 150–200	Rank 200–250	Rank > 250	Strokes gained per round Long total	Long tee	100–150	150–200	200–250	> 250
2010	28	123	29	2	44	16	0.83	−0.08	0.16	0.48	0.12	0.10
2009	1	18	25	1	1	2	2.02	0.53	0.16	0.79	0.43	0.15
2008	1	7	9	1	1	51	2.56	0.60	0.25	1.17	0.40	0.05
2007	1	4	1	1	4	1	2.47	0.81	0.38	0.83	0.30	0.17
2006	1	4	52	1	1	1	2.83	0.91	0.13	0.94	0.62	0.16
2005	1	1	6	16	28	3	2.03	1.09	0.29	0.35	0.14	0.15
2004	5	17	54	2	9	7	1.62	0.53	0.13	0.58	0.24	0.12
2003	2	6	38	2	1	3	2.44	0.87	0.14	0.59	0.59	0.15
2003–2010	1	7	8	1	1	1	2.08	0.70	0.20	0.66	0.35	0.14

Year	Rank Short	Rank 0–20	Rank 20–60	Rank 60–100	Rank Sand	Strokes gained per round Short	0–20	20–60	60–100	Sand
2010	160	169	72	135	173	−0.20	−0.10	0.04	−0.02	−0.11
2009	4	6	1	67	14	0.70	0.25	0.25	0.03	0.17
2008	3	12	1	29	144	0.72	0.23	0.42	0.08	−0.05
2007	24	77	22	22	85	0.41	0.06	0.12	0.10	0.03
2006	16	17	108	18	90	0.45	0.20	0.02	0.12	0.03
2005	98	143	150	67	51	0.09	−0.03	−0.03	0.05	0.08
2004	11	70	2	40	86	0.49	0.08	0.27	0.07	0.04
2003	3	3	34	79	7	0.72	0.41	0.10	0.03	0.17
2003–2010	16	22	8	47	64	0.42	0.13	0.13	0.06	0.07

Year	Rank Putt	Rank 0–6 ft	Rank 7–21 ft	Rank 22+ ft	Strokes gained per round Putt	0–6 ft	7–21 ft	22+ ft
2010	91	58	98	150	0.08	0.11	0.03	−0.06
2009	2	1	40	1	0.99	0.47	0.20	0.31
2008	4	29	12	5	0.85	0.20	0.40	0.25
2007	2	62	3	4	0.80	0.10	0.44	0.26
2006	21	32	58	17	0.49	0.17	0.12	0.20
2005	5	27	15	10	0.70	0.19	0.31	0.20
2004	3	53	2	9	0.96	0.12	0.62	0.22
2003	16	7	32	61	0.55	0.26	0.23	0.07
2003–2010	3	11	4	3	0.70	0.21	0.31	0.19

Table 5: Results, by year, for Tiger Woods are given in the table. Ranks for individual years are based on approximately 220 golfers with at least 30 rounds during each year. An exception was made to show Tiger Woods in 2008, although he only played in three PGA TOUR events. Ranks for 2003–2010 are based on the 299 golfers with at least 120 rounds.

0.89 in his short game, and 0.91 in his putting. His combined results for 2003–2010 show he is the best golfer of his era because of his all-round excellence in every category; his long game contributed 65 percent (2.08/3.20) of his total strokes gained versus the field.

Table 6 shows strokes gained results for selected golfers by year. Steve Stricker was the comeback player of the year in 2006 when his total strokes gained increased from −0.05 to 1.47, moving him from rank 129 to 18. The improvement was almost entirely due to a better long game, with a long-game strokes gained increase from −1.38 to 0.15. He was also the comeback player of the year in 2007; his total strokes gained increased from 1.47 to 1.97, moving him from rank 18 to 5.

Influence of Skill Factors on Golf Scores

Many people claim that the short game and putting are the most important determinants of golf scores. Pelz (1999, p. 1) writes: "60% to 65% of all golf shots occur inside 100 yards of the hole. More important, about 80% of the shots golfers lose to par occur inside 100 yards." Several academic studies have reached similar conclusions. In contrast, strokes gained analysis of PGA TOUR data shows that the long game is the most important factor in explaining the variability in professional golf scores.

For a single golfer, the relative contribution of each skill category can be assessed directly by comparing strokes gained by skill category. Across golfers the relative contributions can be assessed using variance and correlation analysis. Equation (3) is used to estimate μ_i, the mean total strokes gained of golfer i and also the mean strokes gained of long-game shots (μ_i^L), short-game shots (μ_i^S), and putts (μ_i^P). Note that $\mu_i = \mu_i^L + \mu_i^S + \mu_i^P$, and all quantities represent 18-hole round averages estimated using Equation (3). For notational convenience, we drop the golfer subscript i. Then, $\mathrm{Var}(\mu) = \mathrm{Var}(\mu^L) + \mathrm{Var}(\mu^S) + \mathrm{Var}(\mu^P) + 2\,\mathrm{Cov}(\mu^L, \mu^S) + 2\,\mathrm{Cov}(\mu^L, \mu^P) + 2\,\mathrm{Cov}(\mu^S, \mu^P)$ (where each term represents the variance or covariance across golfers). A unique decomposition of $\mathrm{Var}(\mu)$ is complicated because of the covariance terms. However, the covariance terms are quite small and $V \equiv \mathrm{Var}(\mu^L) + \mathrm{Var}(\mu^S) + \mathrm{Var}(\mu^P) \approx \mathrm{Var}(\mu)$. (Using data from 2003–2010 for golfers with at least 120 rounds gives: $\mathrm{Var}(\mu) = 0.50$, $\mathrm{Var}(\mu^L) = 0.35$, $\mathrm{Var}(\mu^S) = 0.06$, $\mathrm{Var}(\mu^P) = 0.08$, $\mathrm{Cov}(\mu^L, \mu^S) = 0.01$, $\mathrm{Cov}(\mu^L, \mu^P) = -0.02$, and $\mathrm{Cov}(\mu^S, \mu^P) = 0.03$.) Therefore, we define the contributions of the long game, short game, and putting to total strokes gained by: $\mathrm{Var}(\mu^L)/V$, $\mathrm{Var}(\mu^S)/V$, and $\mathrm{Var}(\mu^P)/V$, respectively. More variability in a strokes gained category means that golfers have more opportunity to distinguish themselves as better or worse golfers. Using data from 2003–2010 for golfers with at least 120 rounds, the contributions to total strokes gained are 72 percent, 11 percent, and 17 percent for the long game, short game, and putting, respectively. By this measure, *the long game explains more than two-thirds of the variation in total strokes gained*.

Table 7 summarizes correlation results across golfers. When the three broad skill categories are subdivided, approach and tee shots in the 150–200 yard range have the highest correlation (74 percent) with total strokes gained. At the tournament professional level, these skill factors are nearly uncorrelated, as illustrated in Figure 7. The slight negative correlation can be explained by survivorship bias: golfers with a subpar long game need better than average putting (and/or short games) to survive on the PGA TOUR.

Correlation and variability do not equate to importance. If every professional golfer hit every drive 320 yards in the middle of the fairway, then long tee shots would have zero correlation with score, and the variability in long tee strokes gained would be zero. In this example, the golfers do not differentiate themselves with their long tee shots—they are all equally outstanding in this skill category. However, being a good driver of the ball is still important: a golfer who does not hit his drives 320 yards in the fairway will not survive on the tour for long.

Course Difficulty Factors

The estimation of course-round difficulty parameters, δ_j in Equation (3), allows us to rank courses as we ranked golfers. By using individual shot data, course difficulty can be further explained and broken down into difficulty of long-game shots, short-game shots, and putts. Connolly and Rendleman (2012) study a related question on the difficulty of winning a tournament. For handicapping purposes, the USGA rates course difficulty for zero-handicap (scratch) and bogey golfers by methods such as tabulating hole distances and counting the number and severity of

	Rank				Strokes gained			
Year	Total	Long	Short	Putt	Total	Long	Short	Putt
Jim Furyk								
2010	3	26	2	22	2.03	0.90	0.64	0.49
2009	3	31	8	4	2.12	0.80	0.53	0.80
2008	6	17	61	28	1.62	0.98	0.20	0.44
2007	10	14	8	105	1.68	1.02	0.62	0.04
2006	2	3	17	3	2.94	1.69	0.44	0.81
2005	4	6	8	26	2.27	1.32	0.53	0.41
2004	22	33	100	16	1.50	0.83	0.06	0.60
2003	4	13	5	10	2.55	1.31	0.61	0.62
2003–2010	2	10	10	14	2.12	1.13	0.47	0.52
Vijay Singh								
2010	30	3	33	196	1.05	1.42	0.31	−0.68
2009	70	28	61	186	0.40	0.83	0.18	−0.61
2008	4	4	4	177	1.80	1.55	0.63	−0.38
2007	9	6	19	107	1.75	1.25	0.47	0.03
2006	6	6	9	90	2.07	1.43	0.53	0.10
2005	2	5	3	63	2.58	1.77	0.61	0.20
2004	2	1	7	120	2.86	2.28	0.61	−0.03
2003	2	3	6	63	3.06	2.23	0.60	0.23
2003–2010	3	2	5	195	2.05	1.63	0.51	−0.09
Ernie Els								
2010	7	16	37	28	1.75	1.05	0.28	0.42
2009	16	6	23	152	1.37	1.35	0.32	−0.30
2008	25	10	26	190	1.10	1.16	0.40	−0.46
2007	2	2	27	104	2.16	1.74	0.38	0.04
2006	8	15	3	96	1.94	1.11	0.75	0.07
2005	3	3	60	32	2.37	1.80	0.18	0.39
2004	3	4	5	79	2.48	1.63	0.69	0.16
2003	10	8	16	159	1.90	1.67	0.51	−0.29
2003–2010	4	4	15	153	1.86	1.40	0.44	0.01
Phil Mickelson								
2010	10	10	15	118	1.49	1.16	0.39	−0.05
2009	19	23	13	119	1.29	0.92	0.42	−0.05
2008	1	5	8	50	2.25	1.40	0.57	0.27
2007	3	11	4	59	2.06	1.14	0.69	0.23
2006	5	4	32	66	2.13	1.58	0.34	0.20
2005	8	18	7	49	1.82	0.98	0.54	0.30
2004	10	8	25	123	1.79	1.44	0.39	−0.04
2003	46	94	30	54	0.87	0.19	0.39	0.29
2003–2010	5	12	12	95	1.72	1.11	0.47	0.15
Steve Stricker								
2010	1	9	3	15	2.36	1.17	0.64	0.55
2009	2	9	1	56	2.23	1.18	0.75	0.30
2008	14	112	1	26	1.31	0.03	0.83	0.46
2007	5	25	3	25	1.97	0.82	0.73	0.41
2006	18	113	2	20	1.47	0.15	0.82	0.50
2005	129	213	2	8	−0.05	−1.38	0.65	0.68
2004	144	213	9	12	−0.22	−1.41	0.53	0.67
2003	141	188	11	95	−0.35	−0.91	0.53	0.03
2003–2010	22	160	1	19	1.13	−0.02	0.69	0.46

Table 6: The table gives total strokes gained per round for selected golfers, by year. Ranks for individual years are based on approximately 220 golfers with at least 30 rounds during each year. Ranks for 2003–2010 are based on the 299 golfers with at least 120 rounds.

	Total (se) (%)	Long (se) (%)	Short (se) (%)	Putt (se) (%)
Total (%)	100			
Long	79 (2)	100		
Short	54 (4)	6 (6)	100	
Putt	41 (5)	−14 (7)	39 (4)	100
Number of strokes	71.1	32.2	9.8	29.1
Fraction of strokes (%)		45	14	41

	Long game					Short game				Putt		
	Long tee (%)	100–150 (%)	150–200 (%)	200–250 (%)	> 250 (%)	0–20 (%)	20–60 (%)	60–100 (%)	Sand (%)	0–6 (%)	7–21 (%)	22+ (%)
	54	61	74	66	53	50	37	44	33	27	37	40
	(4)	(4)	(3)	(4)	(4)	(4)	(5)	(5)	(5)	(6)	(5)	(5)
Number:	13.9	4.8	7.1	3.2	1.6	4.3	2.1	1.6	1.7	16.0	7.9	5.3
Fraction (%):	19.6	6.7	10.0	4.5	2.3	6.0	3.0	2.1	2.4	22.4	11.1	7.4

Table 7: The tables give correlation results using 2003–2010 data for all PGA TOUR golfers with 120 or more rounds. Top panel: *Total* refers to the total strokes gained per 18-hole round. *Long* refers to the total strokes gained per 18-hole round for shots over 100 yards from the hole. *Short* refers to the total strokes gained per 18-hole round for shots under 100 yards from the hole excluding putts. *Putt* refers to the total strokes gained per 18-hole round for shots on the green. The bottom two rows give the average number of shots and fractions of shots in each category. Bottom panel: correlations of subcategories with total strokes gained. Standard errors (computed with standard bootstrapping procedure) are given in parentheses.

bunkers and other hazards. The USGA's method does not use scores or shot information. To the best of our knowledge, our work is the first attempt at ranking courses using shot data and the first to break down course difficulty by shot categories.

Define the difficulty factors for each course to be the average value of $-\delta_j$ for all rounds played at that course. The negative sign is used so that the most difficult courses are ranked at the top. Table 8

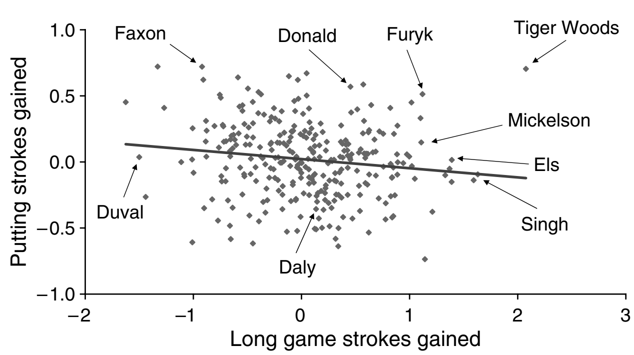

Figure 7: The scatter chart shows putts gained vs. long game strokes gained using 2003–2010 data. Each data point represents the results for a single golfer; a few golfers are used as an illustration. The regression trendline shows a slight negative correlation between the two skill categories (the correlation is −14% with a standard error of 7%).

shows the 10 most difficult and the 10 easiest courses that hosted tournaments during 2003–2010 and had at least 12 rounds of data. The TPC Sawgrass course, host of the Players Championship and famous for the island green on its 17th hole, is ranked as the most difficult course on the PGA TOUR. The strokes gained approach explicitly accounts for the length of the course; thus, courses are rated as more difficult because of factors such as trees, hazards, rough height, firmness, and contours of the greens. The strokes gained approach enables us to see which parts of the course contribute most to its difficulty and to rank courses for difficulty in the long game, short game, and putting. For example, Westchester Country Club is rated as the most difficult course for the short game and putting; Harbour Town Golf Links is rated as the most difficult course in the long-game category.

Effect of the Groove Rule Change

The USGA recently changed its rules regarding the grooves in irons because of the perception that equipment advances in the past decade have made shots from the rough easier: clubs with sharper grooves allow skilled golfers to impart more spin on the ball from the rough and stop the ball closer to the hole. The purpose of the new rule is to "roll back" these

	Rank				Difficulty factors			
Course	Total	Long	Short	Putt	Total	Long	Short	Putt
TPCSawgrass	1	2	3	9	2.41	1.72	0.47	0.23
WestchesterCC	2	19	1	1	1.70	0.16	0.84	0.70
HarbourTownGolfLinks	3	1	27	18	1.69	1.78	−0.12	0.03
MuirfieldVillageGC	4	7	2	10	1.68	0.89	0.64	0.15
BayHillClub	5	3	28	11	1.54	1.58	−0.12	0.09
PebbleBeachGolfLinks	6	6	35	3	1.32	0.91	−0.21	0.62
WestinInnisbrook-Copperhead	7	5	22	12	1.20	1.21	−0.10	0.09
PGANationalChampionCourse	8	4	33	39	0.94	1.38	−0.20	−0.24
QuailHollowClub	9	16	12	4	0.77	0.22	0.05	0.50
TorreyPinesSouthCourse	10	14	25	8	0.48	0.36	−0.11	0.24
Average					1.37	1.02	0.11	0.24
LaCanteraGC	36	36	32	27	−1.07	−0.80	−0.16	−0.11
TucsonNat'lGolf	37	39	24	14	−1.11	−1.07	−0.10	0.07
WarwickHillsG&CC	38	35	37	29	−1.16	−0.77	−0.26	−0.13
MagnoliaGC	39	33	38	38	−1.19	−0.69	−0.29	−0.21
ForestOaksCC	40	37	30	41	−1.33	−0.94	−0.14	−0.25
AtunyoteGolfClub	41	40	40	17	−1.42	−1.13	−0.32	0.03
TPCSummerlin	42	44	6	43	−1.60	−1.67	0.33	−0.26
TPCDeereRun	43	43	23	44	−1.75	−1.29	−0.10	−0.36
SedgefieldCountryClub	44	45	18	19	−1.97	−1.93	−0.06	0.02
En-JoieGC	45	41	45	30	−1.99	−1.17	−0.69	−0.13
Average					−1.46	−1.15	−0.18	−0.13

Table 8: The table provides a ranking of courses by difficulty factors. Ranks are based on the 45 courses that hosted PGA TOUR tournaments and had at least 12 rounds of data during 2003–2010.

equipment advances, so that shots from the rough will have less spin and the rough more of a penalty compared to the fairway. The rule changes were used on the PGA TOUR at the start of the 2010 season.

To determine the effect of this rule change, we estimate benchmark functions representing the average strokes to complete a hole for the fairway and rough for each year. We exclude recovery shots (as described in the *Recovery Shots* subsection) to ensure that the rough benchmark functions are not biased by these shots. Define the rough penalty to be the difference in the average strokes to complete the hole between the rough and fairway at comparable distances to the hole. For example, from 120 yards in the fairway, the average number of strokes to complete the hole is 2.85; it is 3.08 from the rough. The penalty for being in the rough at 120 yards from the hole is an increase of 0.23 strokes. Because the rough penalty varies slightly by distance, we show results for the average rough penalty between 50 and 150 yards from the hole, where the rule is designed to have maximum impact.

Figure 8 shows a decline in the rough penalty from 2003–2009. Surprisingly, it shows a large drop from 2008 to 2009, *prior* to enacting the groove rule change. The rough penalty was unchanged at 0.20 in 2009 and 2010 (with standard errors of 0.004). Because tests of ball spin indicate a measurable impact of the rule change, the small impact on scores is puzzling. Differences in the height, thickness, and moisture of the rough and the firmness of the greens also influence the results; if possible, these factors should be incorporated in the analysis. Another possible explanation is that golfers adapted their swings and strategy to minimize the impact of the change in ball spin. We leave these issues for future research.

Concluding Remarks

The availability of detailed golf-shot data makes it possible to create golf measures that allow consistent comparisons between different parts of the game. Using the starting and ending locations of each shot, strokes gained gives the number of strokes a golfer gains or loses relative to an average PGA TOUR tournament field. Analysis of over eight million shots on the PGA TOUR in 2003–2010 shows that the long

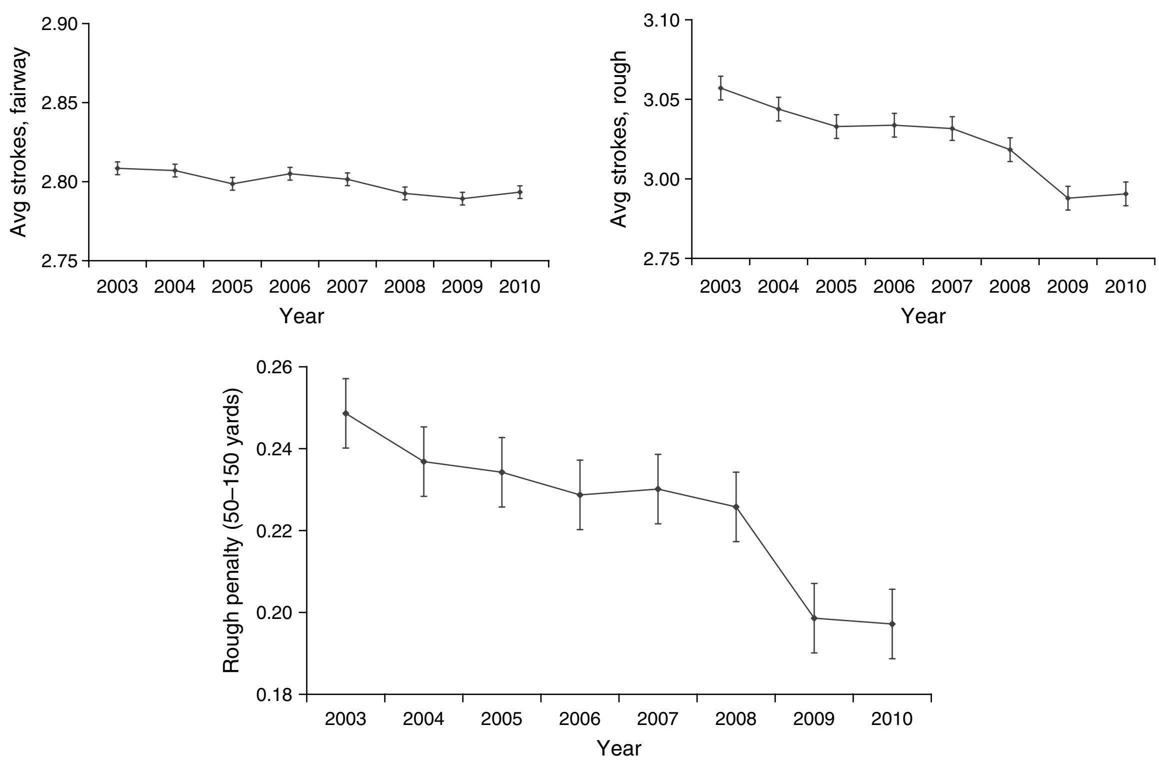

Figure 8: The upper charts show the average strokes to complete the hole from the fairway and rough. The lower chart shows the rough penalty (the difference between the rough and fairway values). All three charts show results by year for shots starting between 50 and 150 yards from the hole. Two standard error bars are shown in each chart (standard errors were computed with a standard bootstrapping procedure).

game accounts for more than two-thirds of the scoring differences between PGA TOUR golfers. Tiger Woods led in total strokes gained, with a gain of 3.20 strokes per 18-hole round. He gained 2.08 strokes (65 percent of the total) in the long game. A preliminary analysis of the impact of the new groove rule for irons that went into effect on the PGA TOUR in 2010 showed, somewhat surprisingly, that it had almost no impact on scores.

Appendix A

This appendix provides the details of the model used for the putting benchmark. The one-putt probability function is based on a simple physical model for putts. Putting skill is modeled using two components: random distance and random direction, both independently distributed normal random variables. The random direction of the putt with respect to the hole is α, with $\alpha \sim N(0, \sigma_\alpha^2)$, so angular that putt errors have a standard deviation of σ_α. The putt rolls a random distance l with $l \sim N(d+t, (d+t)^2\sigma_d^2)$, where d is the initial distance to the hole and t is the target distance beyond the hole (all measured in yards). The standard deviation of the distance a putt rolls, $(d+t)\sigma_d$, is proportional to the intended target distance $(9d+t)$. If $t = 1/2$ yard, the golfer aims to hit the putt 1.5 feet beyond the hole. For the putt to have a chance of finishing in the hole, the angle must satisfy $(|\alpha| \le \alpha_c = \tan^{-1}(r/d))$, where d is the distance to the hole and r is the radius of the hole (2.125 inches). In addition, the distance the putt rolls, l, must be at least d; otherwise, the putt will not reach the hole. If the putt is hit too hard (even if hit straight at the hole) and rolls a distance greater than $d+h$, it will also not result in a holeout, which occurs if the putt rolls a distance l satisfying $d \le l \le d+h$ and is hit with an angle satisfying $|\alpha| \le \alpha_c$.

This model is a generalization of the model in Gelman and Nolan (2002), which only considers putt direction. It is a simplification of Broadie and Bansal (2008), which models distance, direction, and green reading errors, but is not analytically tractable and requires simulation to evaluate. The holeout criterion is used for analytical tractability. Holmes (1991) developed a detailed physical model for holeouts (i.e., the putt finishing in the hole), which Broadie and Bansal (2008) used. This model has few parameters, has a physical interpretation, is analytically tractable, and fits the data well.

The probability of a one-putt, $p_1(d; \sigma_\alpha, \sigma_d, t, h)$, is

$$\begin{aligned} &P(|\alpha| \le \alpha_c)P(d \le l \le d+h) \\ &= P(|\alpha| \le \alpha_c)P\left(\frac{-t}{\sigma_d(d+t)} \le Z \le \frac{h-t}{\sigma_d(d+t)}\right) \\ &= \left(2\Phi\left(\frac{\alpha_c}{\sigma_\alpha}\right)-1\right)\left(\Phi\left(\frac{h-t}{\sigma_d(d+t)}\right)-\Phi\left(\frac{-t}{\sigma_d(d+t)}\right)\right), \end{aligned} \tag{A1}$$

where Z is a standard normal random variable and Φ is the cumulative distribution of a standard normal. Given a set of one-putt data by distance to the hole, an optimization model is solved to find the best-fit parameters, σ_α and σ_d. The model can be fit very quickly because of Equation (A1) and the readily available routines for computing Φ. The best-fit parameters are $\sigma_\alpha = 1.46$ and $\sigma_d = 0.057$ (with the parameters t and h fixed at $t = 1/2$ and $h = 2/3$).

The three-or-more putt probability function is estimated by fitting the equation,

$$p_3(d; a_0, a_1, a_2, a_3) = \frac{1}{1+e^{a_0+a_1 d+a_2 d^2}} + a_3, \tag{A2}$$

for the parameters a_0, a_1, a_2, and a_3. This functional form was chosen because, of the many forms tested, it fit the data well. Four (or more) putts are observed in the professional data, but are so rare that the fit is not affected. The optimization model to find the best-fit parameters is quick to solve.

The average number of putts to holeout benchmark function is now easy to compute using

$$\begin{aligned} J(d) = {} & p_1(d; \sigma_\alpha, \sigma_d, t, h) \\ & + 2(1 - p_1(d; \sigma_\alpha, \sigma_d) - p_3(d; a_0, a_1, a_2, a_3)) \\ & + 3p_3(d; a_0, a_1, a_2, a_3), \end{aligned} \tag{A3}$$

where the condition of starting on the green is implicit. This approach of separately fitting a physical model for one-putts, a statistical model for three-putts, and combining them to give an average-number-of-putts curve is simple, easy to calibrate, and fits the data well. It also allows us to see one-putt, two-putt, and three-putt probability functions and the average-number-of-putts function. Fearing et al. (2010) use a different approach—a statistical model for the one-putt probability function and a gamma distribution fit to the remaining distance of missed putts.

Appendix B

This table in this appendix summarizes the benchmark average strokes-to-complete-the-hole functions from tee, fairway, rough, sand, and recovery positions.

Distance	Tee	Fairway	Rough	Sand	Recovery
10		2.18	2.34	2.43	3.45
20		2.40	2.59	2.53	3.51
30		2.52	2.70	2.66	3.57
40		2.60	2.78	2.82	3.71
50		2.66	2.87	2.92	3.79
60		2.70	2.91	3.15	3.83
70		2.72	2.93	3.21	3.84
80		2.75	2.96	3.24	3.84
90		2.77	2.99	3.24	3.82
100	2.92	2.80	3.02	3.23	3.80
120	2.99	2.85	3.08	3.21	3.78
140	2.97	2.91	3.15	3.22	3.80
160	2.99	2.98	3.23	3.28	3.81
180	3.05	3.08	3.31	3.40	3.82
200	3.12	3.19	3.42	3.55	3.87
220	3.17	3.32	3.53	3.70	3.92
240	3.25	3.45	3.64	3.84	3.97
260	3.45	3.58	3.74	3.93	4.03
280	3.65	3.69	3.83	4.00	4.10
300	3.71	3.78	3.90	4.04	4.20
320	3.79	3.84	3.95	4.12	4.31
340	3.86	3.88	4.02	4.26	4.44
360	3.92	3.95	4.11	4.41	4.56
380	3.96	4.03	4.21	4.55	4.66
400	3.99	4.11	4.30	4.69	4.75
420	4.02	4.19	4.40	4.83	4.84
440	4.08	4.27	4.49	4.97	4.94
460	4.17	4.34	4.58	5.11	5.03
480	4.28	4.42	4.68	5.25	5.13
500	4.41	4.50	4.77	5.40	5.22
520	4.54	4.58	4.87	5.54	5.32
540	4.65	4.66	4.96	5.68	5.41
560	4.74	4.74	5.06	5.82	5.51
580	4.79	4.82	5.15	5.96	5.60
600	4.82	4.89	5.25	6.10	5.70

Table B.1: The table shows the average number of strokes to complete the hole for PGA TOUR golfers from various starting positions. Distance to the hole is measured in yards. Values are estimated using over eight million shots during 2003–2010.

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