Parametric acceleration – the effect of inward pull of the golf club at impact stage

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Abstract

The inward pull motion at the impact stage of the golf swing commonly observed with expert players was investigated in this paper. First, a model of nonconcentrated rotation was studied. It was found that, for a mass rotating around a pivot, if the pivot is moved in the direction opposite to the direction of centrifugal force of the mass, the kinetic energy of the mass could be increased. The increase is a result of the mutual action of the two governing factors of the system, which are the centripetal force and the pull velocity. A special type of equation of motion governs this phenomenon $\ddot{\theta} + \dot{\xi}\sin\theta = 0$, and the parameter in the second term of the left-hand side of the equation $\dot{\xi}$ characterizes its behaviour. The phenomenon is called the parametric acceleration, following the parametric excitation of vibration problems also governed by a similar equation. Second, the golf swing was investigated using the above finding. In the golf swing, the club is accelerated by the hands in a tangential direction. Theoretically, the additional acceleration of the clubhead could be achieved by pulling the club in the radial direction at impact stage, assuming that the centrifugal force of the clubhead is fully developed. To test this idea, an emulation using a modified double-link model was carried out. It was shown that the clubhead velocity could be increased substantially by the inward pull motion of the club at the impact stage, at which point no other means of acceleration is available. Discussions include the actual movement of the body for the inward pull, the efficiency of the pull motion and application to other sports.

Keywords: acceleration, golf, mechanics, parametric, pull, swing

The inward pull motion of the golf club before impact

The starting point of this study was the accurate measurement of the golf club motion during the downswing of several golfers. Measured data are mapped on the *XY*, *YZ* and *ZX* planes in the form shown in Fig. 1. The subject of this example is a low handicap golfer. The time interval between

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each frame is 5 ms and the locus of the instantaneous centre of rotation is also plotted.

The club motions mapped on the *XZ* and *YZ* planes seem normal. The motion of the club mapped on the *XY* plane is interesting because the club is abruptly pulled toward the player well before impact. When these figures are more carefully examined, a sign of the pull is also found in the *YZ* mapping, although it is not distinct. The pull motion in this example starts approximately 30–40 ms before impact and continues up to impact.

The sign of pull can be seen more clearly in the figure where the loci of the club are mapped on the
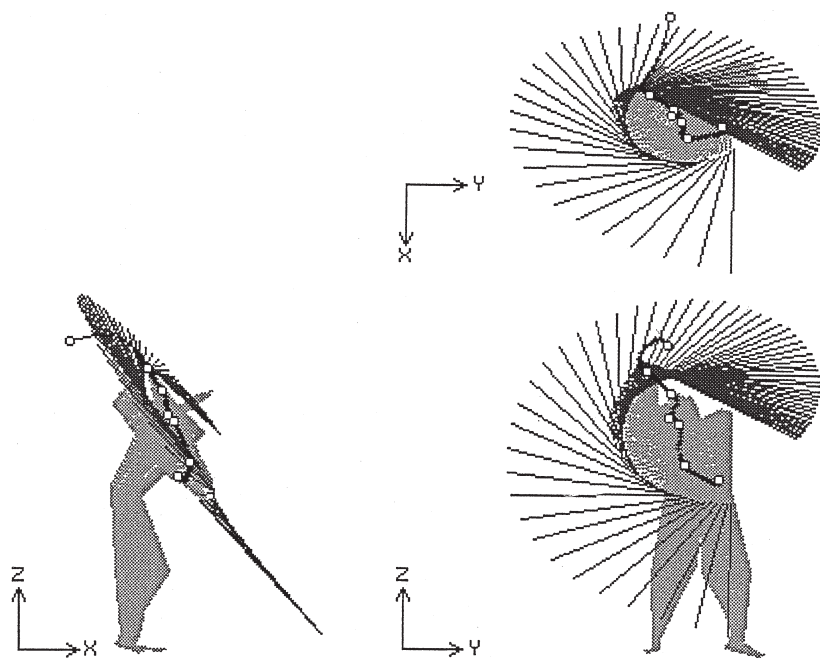
Figure 1 Down swing motion of a club (1 W) projected on *XY*, *YZ* and *ZX* planes (Kaneko, personal communication).
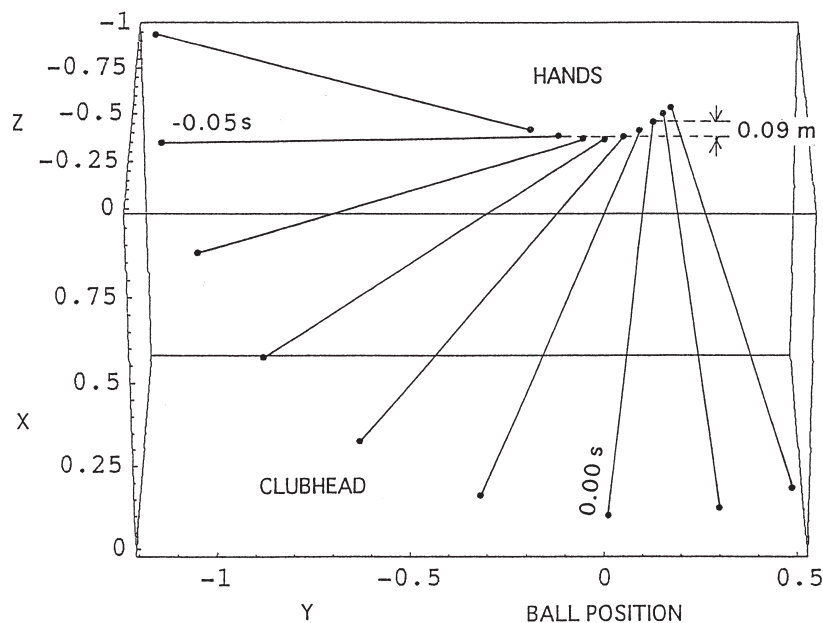

Figure 2 Loci of the club motion (1 W) mapped on the swing plane (30° tilted from the vertical) (Naruo, personal communication).

'swing plane' (30° tilt to the vertical), as shown in Fig. 2. The subject for this case is a ranking professional golfer. As shown in the figure, the curve of the hands position is clearly different from the natural extension of an approximately circular curve of the prior stage. Although the accurate identification is difficult to perform, the hand position pulls inward (to the axis of rotation) about 0.09 m on the swing plane during the impact stage (–40–0 ms) in this instance. It seems that the abrupt shift in this stage could not be explained by the fact that the hands are moving in an approximately

circular arc and are slightly ahead of the ball at impact. It is natural to suspect that there is an additional effect due to an independent cause, i.e. the inward pull motion.

After inspecting the equivalent data on several golfers of varying skill levels, it was found that the pull motion is common to almost every player in varying degrees (similar signs of pull motion are found in Cochran & Stobbs 1968, Fig. 3:3–3:5).

This brings us to the question of the role of the pull motion in the golf swing.

- Such a pull motion might consume a substantial amount of power, because the pull has to resist the large centrifugal force of the clubhead. Is there any gain corresponding to this power consumption?
- Since the direction of pull motion at the impact stage is roughly normal to the direction of the clubhead, it may not contribute to the acceleration of the clubhead.
- The trajectory of the clubhead becomes shallower by the inward pull.
- Instructions on movement of the body, which results in such a pull motion, are abundant in various texts. For instance, McLean (1994) wrote that 'Both hips and shoulders are open to the target line at impact'.

The purpose of this study was to find a rational explanation of the pull motion at the impact stage by analysis and emulation.

Forces applied to the club at the impact stage

In order to study the pull motion at the impact stage, the dynamics of the golf swing without the pull motion has to be known. For this purpose, a base model without the pull motion was investigated. By means of the Working Model software (Knowledge Revolution 1992), a double-link swing model moving on the swing plane was constructed (Fig. 3). Working Model is the motion simulation software that allows engineering problems to be visualized. The program calculates the effects of forces on an object or objects, animates the results and provides data such as force, moment, velocity, acceleration, etc., in digital or graphical form.

The principal structure of the model consists of the arm link (the arms, shoulders and upper torso) and the club link. It is, for the most part, similar to the models that appear in Williams (1969) and Jorgensen (1994). The particular details are shown as follows. The wrist joint is not torque resistant, provided that the minimum cock angle is kept to 90° by the separator. The model is actuated by the torque motor located at the end of the arm link. The time-dependent torque function is determined by trial-and-error simulations in order to satisfy the following constraints: the initial position, the time of downswing, the clubhead velocity vector at impact and the joint power curve presented by
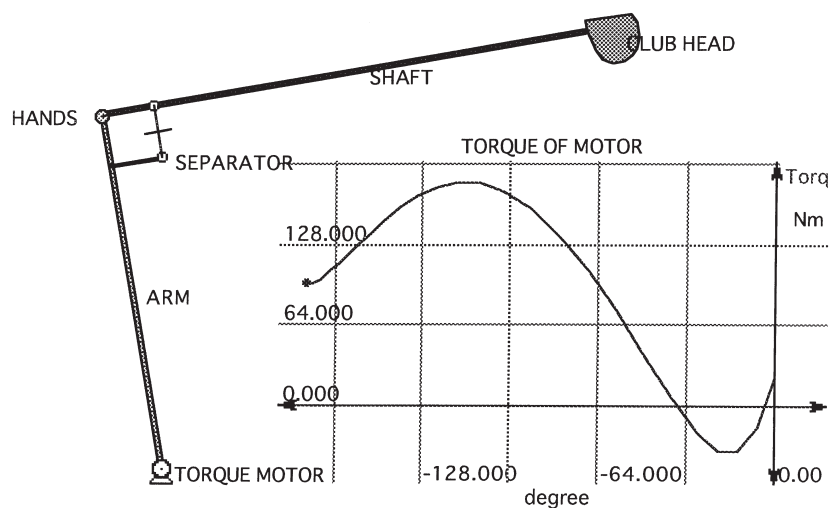
Figure 3 The double-link swing model and applied torque (applied torque ∼ arm rotation).

Kaneko & Sato (1991). The major numerical values of the model are as follows:

> virtual arm: mass = 7.000 kg, length = 0.7 m, initial angle = −170°
> clubhead: mass = 0.200 kg
> shaft: mass = 0.100 kg, length = 1.0 m
> cock angle = 90°
> gravity: 9.8 cos 52° m s^{-2}
> applied torque: 96.6 + 515(t − 0.29) × sin[68.5(t − 0.29)^2] Nm
> downswing time: 0.29 s
> maximum clubhead velocity at impact (t = 0): 46.8 m s^{-1}

Figure 4(a) shows the loci of the club position and the hands force vector. In this figure, the force during the cocked stage is ignored. It is observed that the direction of the hands force is almost parallel to the axis of the club shaft. About 0.04 s before the impact, the direction of the hands force changes rapidly as the shaft turns. At the moment of impact, the hands force vector is virtually normal to the clubhead velocity vector. Figure 4(b) shows the tangential and radial components of the hands force. The data are plotted radially based on the hands position. At the impact, the radial component of the hands force is 414 N, while the tangential component is 7 N. Williams (1969) reported that, for the data of Bobby Jones, the radial and the tangential components were 476

and 0.5 N, respectively, for 50.3 m s^{-1} clubhead velocity.

The full downswing interval is divided into three distinct stages. The first stage is the cocked stage (–0.29∼–0.14 s), during which the arms and club move as if frozen together until the wrists start uncocking. The second is the driving stage (–0.14∼–0.04 s), during which the major part of a golfer's power is put into the rotation of the arms. The third is the impact stage (–0.04–0 s), during which the major part of the kinetic energy is transferred from the arm to the club.

The following important facts are brought to light by inspecting these simulation data.

1 The radial component (414 N) of the hands force at the impact is mainly due to the centripetal force against the centrifugal force of the clubhead and shaft. The gravity contribution is only 2.3 N. (To justify the use of the familiar word 'centrifugal force', the coordinate system that is rotating with the club is used here.)

2 At the impact stage, the clubhead cannot be accelerated by an additional tangential force applied with the hands. As the arm is in the decelerating phase at the impact stage, the additional tangential force would disturb this natural motion. The only necessary action for a golfer at this stage is to apply the pull force (the centripetal force) to balance the large centrifugal
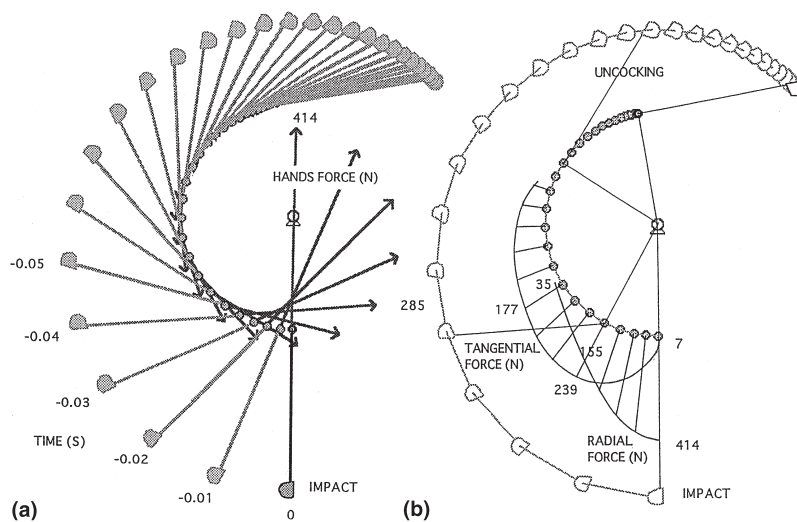

Figure 4 Down swing motion of a club by the double-link model: (a) loci of the club and the hands force vector; (b) tangential and radial components of the hands force.

force, so that he or she can keep the hands in position. The posture of the golfer at this moment is the 'sitting posture'. Williams (1969) wrote, 'This latter pull (radial) merely puts the arm in tension and does nothing to increase clubhead speed, but of course must be there for balance'. If a golfer cannot provide a sufficient pull force to balance the centrifugal force, the club will move a little radially by the force, which means that a work is done by the centrifugal force. As a result, the kinetic energy of the club is consumed and the clubhead velocity is reduced accordingly.

It is interesting to note that if a golfer can supply an extra amount of centripetal force, the club will be pulled inward and the work is done by the centripetal force. With this logic, the pull motion will eventually increase the kinetic energy and velocity of the club. This is exactly the point in dispute in the previous section.

On the basis of the above discussions, we propose that the pull motion at the impact stage increases the kinetic energy of the club and eventually increases the clubhead velocity. In the following sections, this hypothesis is investigated by analysis and emulation.

The theory of parametric acceleration for a system of nonconcentroidal rotation

In order to investigate the hypothesis suggested in the previous section, the dynamics of a swing (a trapeze) provides very useful material. For instance, Toda (1994) showed that a swing is pumped by a mutual action of the centripetal force and the lifting of the centre of gravity of a rider. It is interesting that he proved that the lifting of the pivot of a swing can also pump it. Though pumping a swing is essentially a repetitive motion, the dynamics involved are evidently common to the problem of nonrepetitive motion of the golf swing.

As the first step of the study, the pull motion of a single-link model is analysed. The model consists of a concentrated mass (mass m), a straight bar (length l, zero mass) and a pivot as shown in Fig. 5.

The pivot is movable on the X-axis. It is assumed that the model is initially rotating anticlockwise with a constant velocity and the pivot is stationary at O ($x = y = 0$). The gravity force is not considered here. At some point, when the mass passes through the X-axis, the pivot is moved in the $-X$ direction by a distance ξ. In short, this mechanism represents an essence of the 'pull motion' in the golf swing. If the hypothesis is correct, the mass velocity should increase.

The equations of motion of the single-link model can be written as follows,

$$m\ddot{x} = -F\cos\theta = -F(x + \xi)/l$$
$$m\ddot{y} = -F\sin\theta = -Fy/l \tag{1}$$

where F is the axial force in the bar and θ is the angle between the bar and the X-axis. From Eq. (1), the following relations are obtained.

$$x + \xi = l\cos\theta, \quad y = l\sin\theta$$
$$\dot{x} + \dot{\xi} = -l\dot{\theta}\sin\theta, \quad \dot{y} = l\dot{\theta}\cos\theta$$
$$\ddot{x} + \ddot{\xi} = -l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta \tag{2}$$
$$\ddot{y} = l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta$$

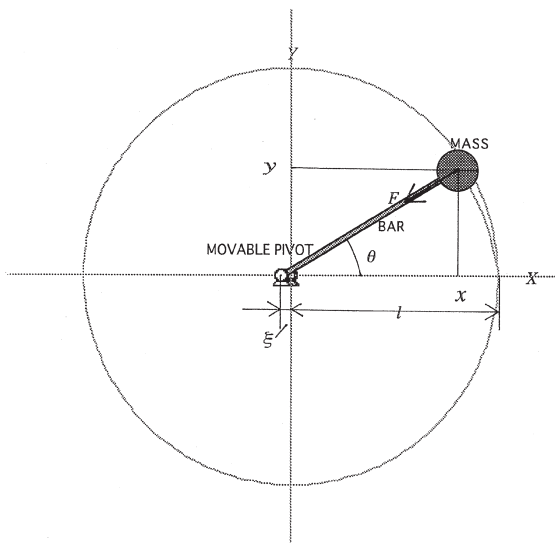Using these relations, Eq. (1) can be written in the following form.

Figure 5 The single-link model with the moving pivot.

$$F = m\left(l\ddot{\theta}\frac{\sin\theta}{\cos\theta} + l\dot{\theta}^2 + \frac{\ddot{\xi}}{\cos\theta}\right)$$
$$F = m\left(-l\ddot{\theta}\frac{\cos\theta}{\sin\theta} + l\dot{\theta}^2\right) \tag{3}$$

Thus,

$$l\ddot{\theta}\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) + \frac{\ddot{\xi}}{\cos\theta} = 0. \tag{4}$$

Then, the equation of motion for θ is obtained,

$$l\ddot{\theta} + \ddot{\xi}\sin\theta = 0$$
or
$$\ddot{\theta} + \xi(t)\sin\theta = 0, \quad \text{where } \xi(t) = l^{-1}\ddot{\xi} \tag{5}$$

which describes the tangential component of the mass trajectory.

The expression of the tensile force F can be obtained from Eq. (3) as follows,

$$F = ml\dot{\theta}^2 + m\ddot{\xi}\cos\theta \tag{6}$$

where the first term of the right-side of the equation represents the centripetal force, and the second term represents the inertia force due to the displacement ξ of the pivot.

The kinetic energy E of the model is

$$E = m\frac{(\dot{x}^2 + \dot{y}^2)}{2} = \frac{1}{2}m\left[(l\dot{\theta})^2 + \dot{\xi}^2 + 2l\dot{\theta}\sin\theta \cdot \dot{\xi}\right]. \tag{7}$$

Differentiating the kinetic energy E with respect to t, the work per second, or the power, done by the tensile force F is obtained as follows,

$$\frac{dE}{dt} = F\dot{\xi}\cos\theta. \tag{8}$$

Equation (8) shows that the work done by the tensile force is the scalar product of the tensile force vector and the velocity vector of the pivot. If the scalar product is positive, the kinetic energy and the mass velocity will increase. The definite integral of the work over a duration of time gives an increase in the kinetic energy of the model. Thus the proposed hypothesis, 'the pull motion at the impact stage increases the kinetic energy of the club

and eventually increases the clubhead velocity', is supported by the analysis of the simplified model.

For a better understanding of this mechanism of acceleration, Eq. (8) can be re-written in the following form,

$$\frac{dE}{dt} = ml\dot{\theta}^2 \cdot \dot{\xi}\cos\theta + m\ddot{\xi}\cos\theta \cdot \dot{\xi}\cos\theta \tag{9}$$

The first term in the right-hand side of Eq. (9) is a product of the centripetal force, the pull velocity and $\cos\theta$. The second term is a product of the inertia force, the pull velocity and $\cos\theta$.

In Eq. (9), if the inertia term $m\ddot{\xi}\cos\theta \cdot \dot{\xi}\cos\theta$ is negligible, the following approximate formula is obtained.

$$\frac{dE}{dt} = ml\dot{\theta}^2 \cdot \dot{\xi} \cdot \cos\theta \tag{10}$$

It is written into a schematic formula as follows.

$$(\text{Power}) \approx (\text{Centripetal force}) \times (\text{Pull velocity})$$
$$\times (\cos\theta) \tag{11}$$

By integrating the above formula during the impact stage, an approximation formula for a gain of the kinetic energy is obtained.

$$(\text{Kinetic energy gain}) \approx (\text{Centripetal force})_{\text{mean}}$$
$$\times (\text{Pull stroke}) \times (\cos\theta)_{\text{mean}} \tag{12}$$

The particular feature of this mechanism of acceleration and energy gain is that it is only generated by the mutual action of factors of the consisting system. The governing factors are the centripetal force and the pull velocity. If there is no centripetal force, the effect of the pull force is simply a translation of the model towards the force direction. It is similar to a situation where a child tries in vain to pump a swing by lifting his or her body, while standing still. A slight rotation of a swing must be there which produces a bit of centripetal force.

It will be beneficial for further discussion to define this mechanism and give it an appropriate name. In the generalized form of the equation of motion (Eq. 5), its behaviour is characterized definitely by the parameter $\xi(t)$. In the case when the parameter is a periodic function of time, it is

well known as the equation for the parametric excitation in the vibration problems. In view of this, the name 'parametric acceleration' is adequate for expressing the mechanism.

Simulation of the parametric acceleration by the single-link model with the moving pivot

In order to confirm the mechanism as well as the theory of the parametric acceleration, a computer simulation of the single-link model with the moving pivot was tested in this section. The Working Model software was used for this purpose.

The model shown in Fig. 6 is in essence a Working Model version of the schematic model in Fig. 5. It consists of a circular disk (mass 0.20 kg, diameter 0.2 m), a straight bar (mass 0.01 kg, length 1.0 m), a torque motor and a linear actuator operating on the X-axis. The pull motion is carried out by the translation of the torque motor by the linear actuator. It is assumed that the disk is initially rotating around the pivot O ($x = 0$, $y = 0$) at an angular velocity of 30 rad s^{-1} (30 m s^{-1} in linear velocity). At some point, when the disk passes the X-axis, the linear actuator starts moving with a constant velocity $\dot{\xi} = 3$ m s^{-1} towards $-X$ axis direction.
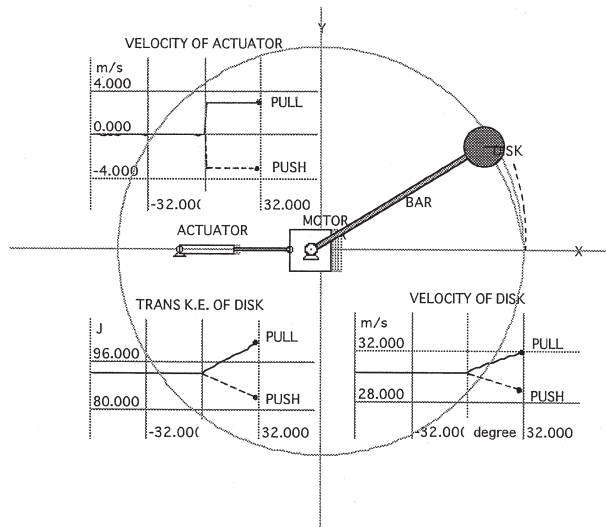

Figure 6 Simulation of the single-link model with pull motion (actuator velocity, kinetic energy, disk velocity ∼ bar rotation).

The resulting trajectory and the relevant data of the model are shown in Fig. 6. Both the translational kinetic energy and the disk velocity are beginning to increase, as the pull motion starts. After 18 ms ($\theta \approx 30°$), the kinetic energy increases from 92.0 to 102.1 J, which is +10.1 J (+ 11%) of the initial value. The disk velocity increases from 30.3 to 31.9 m s^{-1}, which is +1.6 m s^{-1} (+5%) of the initial value. A calculation by the approximate Eq. (12) presents a reasonable estimation as shown in the following.

(Centripetal force)$_{\text{mean}}$
$$= 0.2 \, \text{kg} \times 1 \, \text{m} \times (30 \, \text{rad s}^{-1})^2 = 180 \, \text{N}$$

(Pull stroke) $= 3 \, \text{m s}^{-1} \times 0.018 \, \text{s} = 0.054 \, \text{m}$

$(\cos \theta)_{\text{mean}} = 1$

(Kinetic energy gain) $= 180 \times 0.054 \times 1 = 9.72 \, \text{J}$

(Velocity gain) $= 9.72 \, \text{J} \div (0.2 \, \text{kg} \times 30 \, \text{m s}^{-1})$
$$= 1.62 \, \text{m s}^{-1}$$

For the purpose of comparison, the data for $\dot{\xi} = -3$ m s^{-1}, i.e. the push motion in the direction of centrifugal force, are plotted with dashed lines in these figures. In this case, the work is done by the centrifugal force and therefore the kinetic energy is decreased. One can easily experience the parametric acceleration with a do-it-yourself single-link model consisting of a string (about 1–1.5 m) and an eraser (about 25 g), which is fixed firmly to the string. Rotating the model at a constant velocity, the constant centrifugal force ($-ml\dot{\theta}^2$) can be felt. If the model is pulled quickly inward ($\dot{\xi}$), against the centrifugal force, the power (dE/dt) due to Eq. (9) is put into the system and the increased velocity of the eraser can be felt.

In conclusion, the mechanism and the theory of parametric acceleration are confirmed by the simulation executed by the single-link model. Thus, if a concerned system includes in it a mechanism similar to the above model, it is most likely the mechanism of parametric acceleration will occur. The golf swing is possibly such a system.

Emulation of the parametric acceleration in a golf swing by the modified double-link model with the moving pivot

The next step of this study is to construct a model that can simulate the inward pull motion in the golf swing. The double-link model is not adequate for this purpose because of its limited degrees of freedom (DOF) of motion, as three DOF is the minimum requirement for realizing such a pull stroke at the impact stage.

An idea adopted in this paper is to modify the double-link model by installing a linear actuator as an additional element or link. The modified double-link model is show in Fig. 7. The linear actuator operating on the Y-axis is added to the torque motor, i.e. the centre of rotation of the base model (Fig. 3). By moving the centre of rotation, the equivalent pull stroke in the golf swing can be achieved. The resulting modified double-link model becomes a system of three DOF. It does not directly simulate the actual pull motion by golfers, but emulates the equivalent pull stroke. It is assumed that the linear actuator operates during the impact stage (–0.04–0 s), and that the displacement follows the time-dependent function.

$$\dot{\xi} = 100(t + 0.04), \quad -0.04 \leq t \leq 0 \tag{13}$$

The actuator starts –0.04 s before the impact with a constant acceleration and the total displacement reaches 0.08 m at the impact.

Figure 8 shows the trajectories of the club with the hands force vector, the velocity of clubhead and the velocity of the linear actuator. A slight change in the impact point is ignored in the following discussion. The curves drawn in a broken line refer to the base model. It is shown that the clubhead velocity distinctly increases as the pull motion
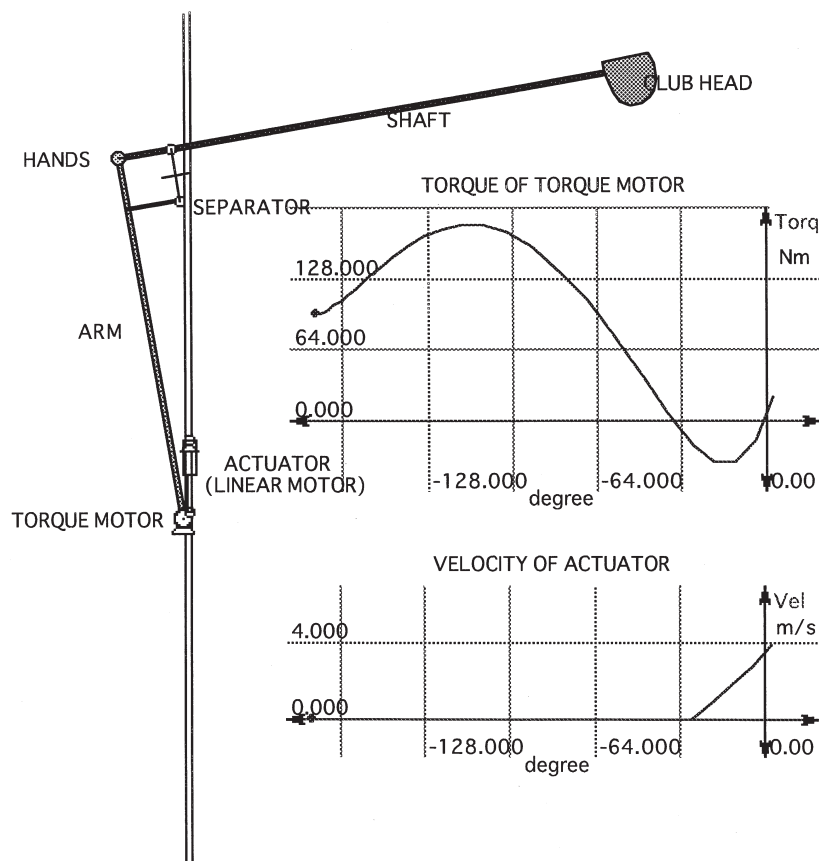

Figure 7 The modified double-link model with the moving pivot (applied torque, applied velocity ~ arm rotation).

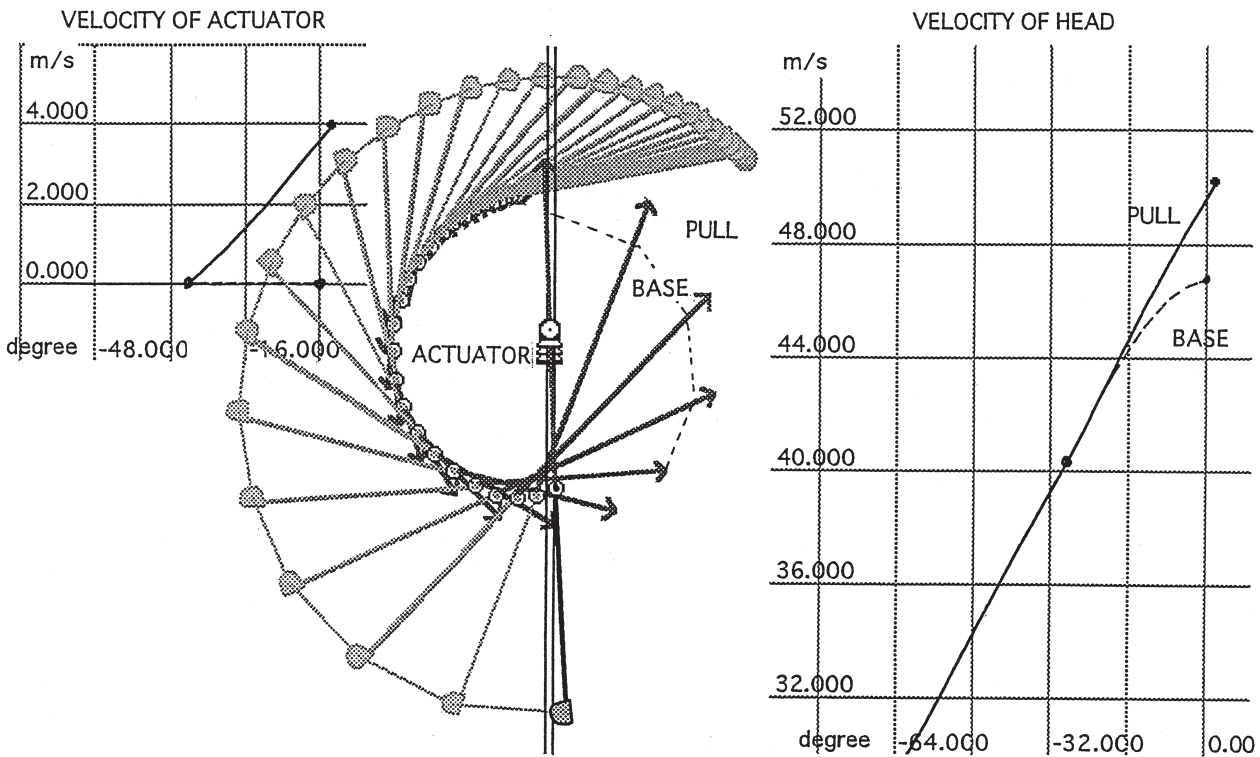VELOCITY OF ACTUATOR

VELOCITY OF HEAD

Figure 8 Emulation by the modified double-link swing model with pull motion (hands force vector, actuator velocity, head velocity ~ arm rotation).

starts. At the same time, a considerable increase in the hands force vector is observed in the figure. Some of the numerical data are shown in Table 1. The increase of the clubhead velocity, 3.3 m s^{-1}, is a relatively large value for the moderate pull stroke of 0.08 m. This result is ample proof that an application of the pull motion substantially increases the kinetic energy and the clubhead velocity.

To supplement the above emulation result, some of the analytical formulae used for the single link model may be useful. A rough estimation of the present example using Eq. (12) is as follows.

Table 1 Effect of pull motion on kinetic energy and velocity of clubhead

Values at impact	Without pull motion	With pull motion
Kinetic energy (J)	161.4	187.2 (+25.8)
Clubhead velocity (m s^{-1})	46.8	50.1 (+3.3)
Hands force (N)	414.1	484.6 (+70.5)

$$(\text{Centripetal force})_{\text{mean}} \approx 400\,\text{N}$$
$$(\text{Pull stroke}) = 0.08\,\text{m}$$
$$(\cos\theta)_{\text{mean}} \approx 0.9$$
$$(\text{Kinetic energy gain}) \approx 400 \times 0.08 \times 0.9 = 28.8\,\text{J}$$

This is a satisfactory estimation of the value 25.8 J of emulation (Table 1). Thus, Eqs (11) and (12) are approximately valid for the golf swing and can be used for supplementing the numerical emulation. In addition, the role of each factor on the acceleration can be understood more clearly by using these formulae.

Another effect of the pull motion is the change in the trajectory of the club (Fig. 9). The trajectory of the clubhead at the impact stage is much shallower than in the model without pull motion. In other words, the clubhead travels straight longer at the impact stage than in the model without pull motion.
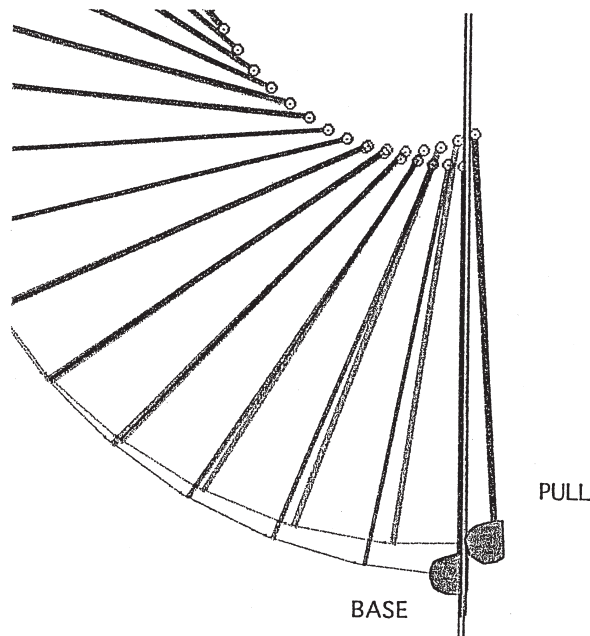
Figure 9 Trajectory of the club with and without pull motion.

In conclusion, the emulation as well as the analytical approximation verify the parametric acceleration at the impact stage of the golf swing.

Discussion

In this paper, a rational explanation of the pull motion by the mechanism of parametric acceleration was investigated. Within the scope of this paper, the obtained explanation has been theoretically justified. This will be discussed further in this section.

Execution of the pull motion

It is relevant to see how an actual golfer goes about achieving this movement. The author's image of the swing executed by an expert player is shown in Fig. 10. The sketch shows silhouettes of three key positions, marked −2, −1 and 0 (impact), from the upper-front viewpoint.

- The trajectory of the hands position starts to deviate from an approximately circular curve at the position −1. There is a substantial amount of hands pull between the positions −1 and 0.
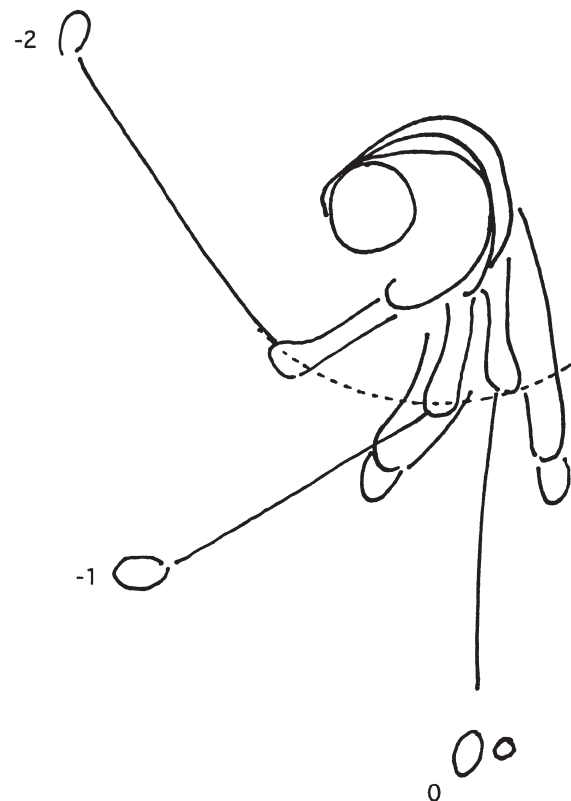

Figure 10 Author's image on the swing by expert golfers (viewpoint: upper front side).

- The pull might be closely relating to the aggressive shift of the left shoulder.
- At first glance, the swing can be modelled well by a three-link model consists of the shoulder, the left arm and the club during this period. However, the major contribution due to lifting of the left leg might be concealed in this view.
- It is safe to say that the coordination of the quick rotation of shoulder/waist and the quick lifting of the left side of the body generates the inward pull motion.

Power of parametric acceleration

One of the important points of the mechanism of parametric acceleration is that additional power can be applied to increase the clubhead velocity at the impact stage, where no other means of doing so are

available. In the current texts on golf swing, the observed pull motion is generally expressed as the passive motion that is necessary for concluding the stroke or controlling the shot. Such an expression is not supported by this study. The inward pull motion is the positive motion that provides a source of power at the crucial stage of impact.

The necessary power depends on the mutual action of the centripetal force of the club and the pull velocity described approximately by Eq. (11).

$$(\text{Power}) \approx (\text{Centripetal force})$$
$$\times (\text{Pull velocity}) \times (\cos \theta)$$

The centripetal force represents the golfer's effort accumulated during the down swing. It should be noted that the centripetal force is approximately proportional to the square of the clubhead velocity. The pull velocity represents the effort of pull at the impact stage. It is the quickness and not the magnitude of pull that is effective for the parametric acceleration. The cosine term depends on the

relative direction of the centripetal force and the pull velocity, and therefore it is an index of the effectiveness of the pull motion on the power.

As already mentioned, the tendency is that the horizontal x-component of the pull motion is usually larger than the vertical z-component. Thus the pull motion vectors are not necessarily on the swing plane, but in most cases deviate from it to a certain extent. In general, it is a function of x, y, z and t. It can only be said that the component of the pull motion on the direction of the tensile force influences the power. A three-dimensional analysis of the problem is now required.

Parametric acceleration in other sports

The mechanism of parametric acceleration may be found in various sports, such as tennis, baseball and hockey. One of many sports that might bear close relation to parametric acceleration is the hammer throw. In Eq. (5), if the displacement of the pivot is
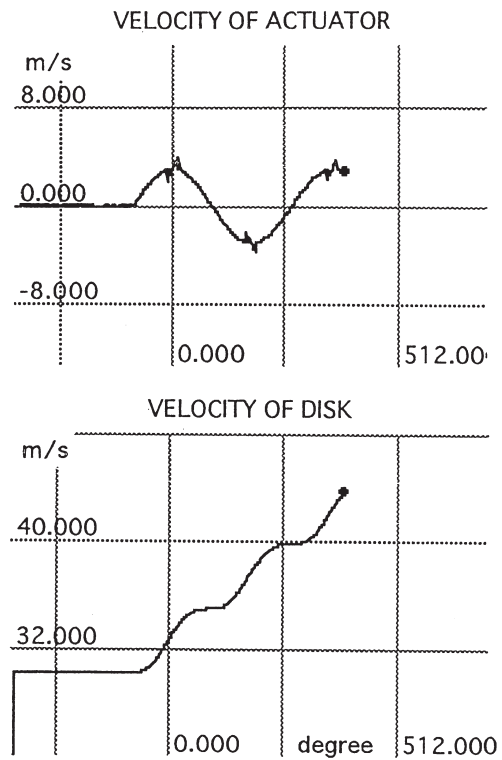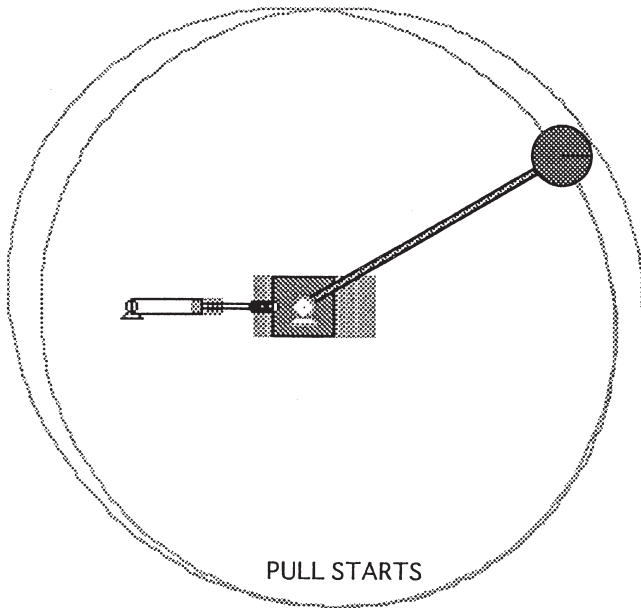

Figure 11 Parametric excitation of the single-link model by periodic pull motion (actuator velocity, disk velocity $\sim$ bar rotation).

replaced with a periodic function of θ, such as $\xi(\theta)$, Eq. (5) becomes:

$$\ddot{\theta} + \xi(\theta)\sin\theta = 0 \qquad (14)$$

This differential equation exhibits quite particular properties depending heavily on periodicity of the function $\xi(\theta)$. For the single-link model of Fig. 6, if an appropriate function is given, the mass velocity periodically increases as shown in Fig. 11.

In this example, the given function is $\xi(\theta) = 3\cos\theta$, so that the pull motion is maximized at $\theta = 0°$, 180°, 360°, …. It is worth mentioning that the velocity curve of this simulation closely resembles the velocity curve of the hammer throw reported by Wakayama *et al.* (1992). A similar repetitive type of amplification is known as the parametric amplification in electronic circuit theory and the parametric excitation in vibration problems, respectively.

A future research area on the subject should be the thorough measurement and analysis of swing data on selected expert players. The quantitative measurement of pull strokes, including their magnitude, direction and time-derivatives, are needed.

Conclusion

The inward pull motion at the impact stage of the golf swing was investigated in this paper.

The theory of parametric acceleration on a system of nonconcentrated rotation was developed for this purpose. For a mass rotating around a pivot, if the pivot is moved toward the direction opposite to the direction of centrifugal force of the mass, the kinetic energy of the mass can be increased. The increase is a result of the mutual action of the two governing factors of the system, i.e. the centripetal force and the pull velocity. Application of this theory to the golf swing revealed that the clubhead velocity could be increased by the inward pull motion of the club at the impact stage, during which no other means of acceleration are available. It was confirmed by the emulation of the modified double-link model of the golf swing and the comparison with the observed data.

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