INSTITUTE OF PHYSICS PUBLISHING REPORTS ON PROGRESS IN PHYSICS

Rep. Prog. Phys. **66** (2003) 131–171 PII: S0034-4885(03)31567-2

The physics of golf

A Raymond Penner

Physics Department, Malaspina University-College, Nanaimo, British Columbia, V9R 5S5, Canada

Received 31 October 2002
Published 20 December 2002
Online at stacks.iop.org/RoPP/66/131

Abstract

An overview of the application of physics to the game of golf is given. The golf swing is modelled as a double pendulum. This model and its variations have been used extensively by researchers in determining the effect that various swing parameters have on clubhead speed. These results as well as examples of three-link models are discussed. Kinematic and kinetic measurements taken on the recorded downswings of golfers as well as force measurements are reviewed. These measurements highlight differences between the swings of skilled and unskilled golfers.

Several aspects of the behaviour of a golf ball are examined. Measurements and models of the impact of golf balls with barriers are reviewed. Such measurements have allowed researchers to determine the effect that different golf ball constructions have on their launch parameters. The launch parameters determine not only the length of the golf shot but also the behaviour of the golf ball on impact with the turf. The effect of dimples on the aerodynamics of a golf ball and the length of the golf shot is discussed. Models of the bounce and roll of a golf ball after impact with the turf as well as models of the motion of a putted ball are presented.

Researchers have measured and modelled the behaviour of both the shaft and the clubhead during the downswing and at impact. The effect that clubhead mass and loft as well as the shaft length and mass have on the length of a golf shot are considered. Models and measurements of the flexing of the shaft as well as research into the flexing of the clubface and the effects of its surface roughness are presented. An important consideration in clubhead design is its behaviour during off-centre impacts. In line with this, the effects that the curvature of a clubface and the moments of inertia of the clubhead have on the launch parameters and trajectory of an off-centred impacted golf ball are examined.

Contents

1. Introduction

The game of golf, in one form or another, has been around for over five hundred years. It can be assumed that even during the very early stages of the game, players, especially those with a scientific inclination, were curious as to the behaviour of the ball and the clubs, and experimented on ways to improve their performance. Over the years, changes to the equipment, as well as the golf swing, have been based more on trial and error than on any application of scientific principles. It has only been over the last several decades that science, and physics in particular, has been used in a significant way to understand and improve the performance of golfers and their equipment. In fact, it can be argued that the scientific study of the game undertaken by the Golf Society of Great Britain in 1968, with their findings presented by Cochran and Stobbs (1968) in their book *Search for the Perfect Swing*, is the starting point of the scientific development of the game of golf. Since then a myriad of papers in a wide range of journals have appeared on the subject. Books by Daish (1972) and more recently by Jorgensen (1994), and Werner and Greig (2000) have all provided detailed analyses of different aspects of the game. In 1990, the first World Scientific Congress of Golf was held in St Andrews, Scotland, with topics ranging from biomechanics to ball dimple patterns. This Congress has met every four years since, and its published proceedings (Cochran 1990, Cochran and Farrally 1994, Farrally and Cochran 1999) have been a major source for this review.

The goal of much of the scientific research into the game and its equipment has been to improve the performance of golfers, professional and amateur alike. This research has not been totally altruistic as golf companies compete for larger portions of the golfing world's economic pie. However, some research has also been carried out solely in order to understand the physics behind some particular phenomena, and few sports provide as many physics problems as does the game of golf.

This review paper will present many examples of the research that has been carried out in the field. Certainly, not all of the research that has found its way into the many journals, books and conference proceedings is included in this paper. Also, for the work that is reported, typically only a small part of the given research is presented. The author apologizes for the omissions; however, it is hoped that the theories, models and results that are presented will give the reader a broad overview of the field. The research has been divided into three main areas: the physics of the golf swing, the physics of the golf ball and the physics of the golf club. Although some of the research overlaps more than one of these areas, this division has allowed the author to maintain some order in the manuscript.

2. The physics of the golf swing

There is no aspect of golf that has been discussed, analysed and advised upon more than the golf swing. The most comprehensive scientific study of the golf swing is still that which was undertaken by the Golf Society of Great Britain and presented by Cochran and Stobbs (1968). In this study the golf swing is modelled as a double pendulum system, a model that later researchers have used extensively to analyse the swings of both skilled and unskilled golfers. The first part of this section on the physics of the golf swing will focus on the double pendulum swing model, and its variations, along with its use in determining ways to increase the clubhead speed. The use of three-link swing models in the analysis of golf swings will then be discussed. Following this, some of the kinematic, kinetic and force measurements that have been taken on golfers, both skilled and unskilled, and their relationship to the swing models, will be presented.

2.1. Modelling the golf swing

2.1.1. Double pendulum model. Figure 1 shows the motion of the arms and golf club during a typical downswing. The primary model that golf researchers have used to analyse this motion is based on a double pendulum or double link system. The two links that are used in this model are included in figure 1. The upper link represents the golfer's shoulders and arms, which rotate about a central hub corresponding roughly to a point between the golfer's shoulders. The lower link represents the golf club, which rotates about a point or hinge located at the centre of the hands and wrists of the golfer. The basis for the double pendulum model stems from the modern practice of keeping the left arm (for a right-handed golfer) straight during the downswing. In the model the backswing is normally ignored and the two links begin in the stationary position that is shown in figure 1(*a*). The angular position of the lower link, θ, at the top of the backswing is referred to as the backswing angle while the angular position of the upper link, with respect to the lower link, is referred to as the wrist-cock angle, α. During the downswing the two links are taken to rotate in a single plane that is inclined to the vertical. In the basic version of the model the hub is taken to be fixed in position and all the golfer's efforts in rotating his or her hips, trunk and arms are equated to a single couple, τ_0, applied at this central pivot. In addition, this couple is often modelled to be constant over the duration of the downswing.

Measurements of the angular positions of the arms and clubs of skilled golfers, such as those made by Budney and Bellow (1979, 1982), indicate that the wrist-cock angle stays approximately constant during the first half of the downswing. The downswing in the double pendulum model has, therefore, typically been divided into two distinct stages. During the first stage of the downswing the wrist-cock angle is fixed and the double pendulum system rotates as one. The value of the wrist-cock angle during this first stage is primarily determined by the ability, or inability, of a golfer to cock his or her wrists. The couple exerted by the hands and wrists, τ_h, on the club or lower link, which is required to maintain this wrist-cock angle, will initially be positive for the orientation shown in figure 1(*a*). During this time the hands and wrists are basically behaving as a stop in preventing the club from falling back towards the golfer. As the downswing proceeds the centrifugal force acting on the lower link will increase and the couple required by the hands and wrists to maintain the wrist-cock angle will subsequently decrease. When the required couple drops to zero the lower link will, unless prevented, begin to swing outwards. This is referred to as a natural release. Jorgensen (1970)

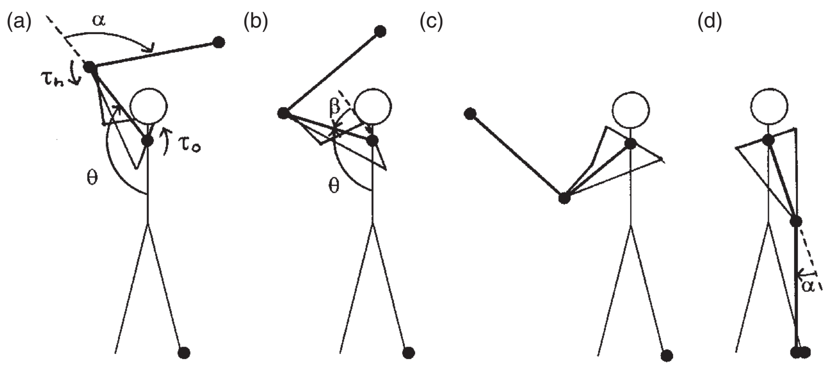

Figure 1. The double pendulum model of the golf swing; (*a*) at the beginning of the downswing; (*b*) at the release point; (*c*) after releasing the club and (*d*) at impact.

showed that for the basic double pendulum swing model the required hand couple drops to zero when the upper link has swung through an angle, given by β in figure 1(*b*), of $I_L/2L_U S_L$, where S_L and I_L are the first and second moments of the club about the centre of the golfers hands and L_U is the length of the upper link. For a typical golf swing and driver this corresponds to an angle of approximately 47°. The angular position of the upper link at the point where the club swings out, θ in figure 1(*b*), is referred to as the release angle. An expert golfer normally swings the club in such a manner that the wrist-cock angle stays fixed beyond the natural release point. If the hands and wrists continue to hold the club at a constant wrist-cock angle after the natural release point, the couple they exert must become increasingly negative. A typical downswing lasts on the order of 200 ms and the first stage is normally modelled as lasting from 100 to 150 ms.

The second stage of the downswing begins when the club swings outwards, either naturally or with a delayed release, and ends when contact with the ball is made. The hands and wrists are normally modelled as exerting no couple during this stage. In the case of a delayed release, the club will swing out much more rapidly than is the case for a natural release due to the larger centrifugal force that is acting on it. As is shown in figure 1(*d*), the golf ball is typically positioned ahead of the hub so that at impact the final wrist-cock angle, α, is not equal to zero.

Daish (1972), Lampsa (1975) and Jorgensen (1970, 1994) derive the equations of motion for the basic double pendulum model using the Langrangian approach. Expressions for the kinetic energies of the two links are determined and the potential energy of the system is equated to the work done by the couples, which are exerted at the hub and at the hands. In the simplest case, where the effects of gravity are neglected, Daish (1972) derives the following equations of motion:

$$A\ddot{\theta} + B\ddot{\phi}\cos(\phi - \theta) - B\dot{\phi}^2 \sin(\phi - \theta) = -\tau_0 + \tau_h \tag{1a}$$

and

$$B\ddot{\theta}\cos(\phi - \theta) + B\dot{\theta}^2 \sin(\phi - \theta) + C\ddot{\phi} = -\tau_h, \tag{1b}$$

where the generalized coordinates, ϕ and θ, are the angular positions of the two links with respect to the vertical. The angular position of the upper link, θ, is as shown in figure 1(*a*), while ϕ, the angular position of the lower link, is given by $\theta + \alpha$. The constants A, B and C, are functions of the mass, the length and the first and second moments of the two links. During the first stage of the downswing the wrist-cock angle, $\phi - \theta$, is held fixed and the general equations reduce to

$$\ddot{\theta} = \ddot{\phi} = -\frac{\tau_0}{I}, \tag{2}$$

where I is the moment of inertia of the whole system about the hub.

The orientation of the club at impact, in the two-stage double pendulum model, will depend on the magnitude of the couple applied at the hub, as well as the backswing, initial wrist-cock, and release angles. In figure 1(*d*) the club is in the vertical position at impact, which, in general, is what would be wanted. Figure 2 shows an example where the values for the hub couple, release angle and wrist-cock angle, are the same as were used for figure 1, but the backswing angle is greatly reduced. In this case the hands will be ahead of the clubhead at impact and the ball would be missed or at best miss-hit. In the case of an increased backswing angle, with the other parameters fixed, the model shows that the clubhead will lead the hands at impact. Timing is, therefore, crucial in the model, as it is with a real golf swing, and changing one parameter in the model will normally require another parameter to be changed if the clubhead is to make solid contact with the golf ball.

Although, the two-stage double pendulum model is a good approximation to the downswing of skilled golfers, researchers have found that better agreement can be obtained if

Figure 2. The position of the two links at impact in the case of a reduced backswing.

the constraint of a fixed hub is removed. Jorgensen (1994) did a detailed study of the swing of a professional golfer and found that including a lateral shift of the hub was necessary in order to get good agreement between modelled and experimental positions and speeds of the club, obtained from a stroboscopic photograph during the downswing. Golfers are typically advised to keep their centre or hub still during the swing, but photographs of expert golfers often indicate that they do tend to shift towards the ball during their downswing. Jorgensen modelled this shift as an initial positive acceleration of the hub towards the ball followed by a negative acceleration. Figure 3 shows the position of the two links as determined by Jorgensen's swing model. A constant value for the hub torque of 77.3 N m, along with a backswing angle of 166°, an initial wrist-cock angle of 124°, and a release angle of approximately 110° was used to generate this swing. The lateral acceleration of the hub was taken to be 14.4 m s^{-2} ($1.47g$) during the first 160 ms of the downswing followed by a negative acceleration of 15.4 m s^{-2} ($1.57g$). The parameters used in Jorgensen's double swing model cannot be considered as unique, although the agreement that was obtained between the modelled clubhead speeds and the measured ones was found to be excellent.

Jorgensen (1994) also discusses the energy transfer during the modelled downswing. The work done on the double pendulum system during the downswing by the hub couple, τ_0, which is taken to be constant, will simply be $\tau_0 \times \beta$. In Jorgensen's model, work is also done on the system by the lateral shift of the hub and by the gravitational force. Figure 4 shows the resulting time evolution of the kinetic energy of the two links in the swing model. As is shown in the figure, the kinetic energy of the upper link first increases, reaching a maximum halfway through the swing, and then decreases through the rest of the downswing. The kinetic energy of the lower link, the club, increases throughout the downswing. This figure clearly shows that kinetic energy is transferred from the upper link to the lower link during the downswing and that the slowing of the hands before impact, which some golf experts advise to avoid, is a natural consequence of this. For this particular swing model, 71% of the total kinetic energy of the system at impact comes from the work done by the applied couple at the hub, 13% comes from the decrease in gravitational energy, and 16% from the work done on the system by the shift of the hub towards the target.

Reyes and Mittendorf (1999) use a variation of Jorgensen's lateral shift to model the swing of a long distance driving competitor. In this model the lateral shift is due to a constant force that is applied at the hub over the duration of the downswing. They found that with a shift force of 40.0 N applied at the hub, along with a hub couple of 81.3 N m, their model matched well with Jorgensen's modelled swing. They then found that fitting this model to the swing of

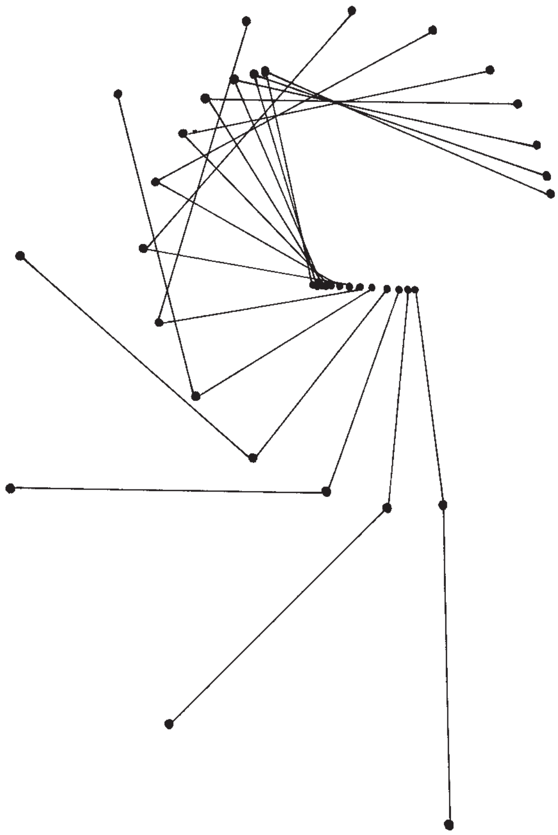

Figure 3. Jorgensen's double pendulum model of the downswing. Adapted from figure 3.3 in Jorgensen (1994).

the long distance driving competitor, by matching modelled and experimental clubhead speeds at impact, required a hub couple of 94.9 N m, a shift force of 89.0 N, along with a backswing angle of 180° and an initial wrist-cock angle of 90°.

Miura (2001) presents a double pendulum swing model in which an upward force is applied at the hub just prior to impact. This inward pull motion at the impact stage has been observed with some expert players. Miura found that a constant upward acceleration of the hub applied to the model during the final 40 ms before impact resulted in good agreement between the modelled and measured hand positions of a low-handicap golfer.

Lampsa (1975) did not use the standard two-stage model of the downswing but instead used optimal control theory to determine how τ_0 and τ_h should vary during the downswing so that the clubhead speed would be a maximum at impact. Both the peak values of the couples and the total time of the downswing were fixed in the analysis. The optimal τ_0 and τ_h were both found to increase approximately linearly through the downswing. An interesting result of the analysis is that not only was the clubhead speed found to evolve in basically the same manner as is found with the standard two-stage model, with a constant τ_0, but the calculated wrist-cock angle was found to stay approximately constant over the first half of the downswing. This would imply that applying smoothly increasing torques at the hub and the hands can give the same apparent constant wrist-cock angle during the initial part of the downswing that is observed with expert golfers and that is built into the two-stage double pendulum model.

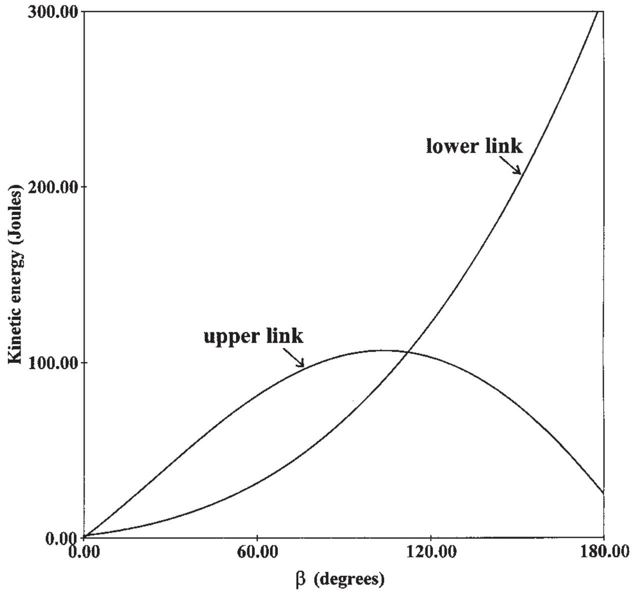

Figure 4. The evolution of the kinetic energy of the two links in the double pendulum swing model. Adapted from figure 5.1 in Jorgensen (1994).

2.1.2. Increasing clubhead speed. Researchers have used the double pendulum model to determine the effect that the various swing parameters have on the impact speed of the clubhead. The clubhead speed at impact is the primary factor in determining the length of a drive and, as a rough approximation, each percentage gain in clubhead speed will result in a corresponding percentage increase in drive distance. In varying the parameters in the swing model there is, however, the constraint that the golf club needs to be approximately vertical at impact if solid contact with the golf ball is to be made. This will typically mean that if one parameter is varied it will be required to vary another parameter in order to restore the timing.

As would be expected, increasing the backswing angle or the initial wrist-cock angle in the double pendulum model is found to lead to a greater clubhead speed at impact. For example, Reyes and Mittendorf (1999) found that for their swing model increasing the backswing angle from 180° to 190°, a 5.6% increase, resulted in clubhead speeds increasing by approximately 3.1%. However, the model also shows (Mittendorf and Reyes 1997) that changing the backswing angle or initial wrist-cock angle has a significant effect on timing. For example, increasing the backswing angle while keeping the other swing parameters fixed would, in general, result in the clubhead leading the hands of the golfer at impact while increasing the initial wrist-cock angle results in the hands leading the club.

The release angle is one swing parameter whose effect on clubhead speed is not as obvious. As has been stated, measurements have shown that expert golfers normally maintain the initial wrist-cock angle beyond the natural release point. Cochran and Stobbs (1968), Jorgensen (1970, 1994), and Pickering and Vickers (1999) have all considered the effect of delaying the release of the club in the double pendulum swing model and have found that, in general, this will result in a greater clubhead speed at impact. For example, Pickering and Vickers found that for their swing model, reducing the release angle from 132°, which corresponds to a natural release, down to 90°, resulted in an increase in the clubhead speed at impact of approximately 3%. Delaying the release of the club results in greater clubhead speed at impact by keeping the club closer to the hub during the downswing so that the first and second moments of the double pendulum system about the hub will be reduced. Delaying the release will, however, also have an effect on the orientation of the club at impact, with the hands, in general, leading the clubhead at impact.

Pickering and Vickers (1999) also investigated the effect of positioning the golf ball forward of the hub point. This is normally the practice of golfers and is included in figure 1(*a*). One benefit of doing this is that for a properly timed swing the clubhead speed will be increasing throughout the downswing. Positioning the ball forward and thereby delaying the impact will, therefore, result in a greater clubhead speed at impact. Pickering and Vickers specifically determined the ball position that would result in the maximum horizontal component of clubhead velocity at impact in their swing model. They found that in the case of a drive and with a natural release, the optimal ball position was 0.226 m forward of the hub position. This resulted in an increase of 1.3% in the clubhead speed compared to having the ball placed in line with the hub. For a delayed release the effect is slightly greater. For example, at a release angle of 110° they found that the optimal ball position is 0.249 m forward of the hub and the corresponding increase in clubhead speed at impact is 1.6%. Pickering (1998) also considered the optimal ball position for other clubs and found that for the shorter and more lofted clubs, the ball should be placed closer to the hub position to maximize the horizontal component of the clubhead velocity at impact.

Certainly, increasing the couple applied at the hub would also be expected to increase the clubhead speed at impact. Jorgensen (1994) found that a 5% increase in τ_0, in his swing model, leads to an increase in the clubhead speed at impact of 1.7%. Reyes and Mittendorf (1999) found that increasing τ_0 by 29%, in their swing model, resulted in the clubhead speed increasing by 8.5%. The gain in clubhead speed is, therefore, not commensurate with the increase in the applied couple at the hub. Not only will the magnitude of the hub couple affect the clubhead speed at impact, but also the way the hub couple evolves during the downswing. As has been discussed, Lampsa (1975) found that the optimal hub couple, in terms of maximizing clubhead speed, increased approximately linearly throughout the downswing.

Jorgensen (1994) also considered the effect of exerting a positive couple at the hands and wrists throughout the downswing, in addition to the couple required to maintain the constant wrist-cock angle during the first stage. Surprisingly it was found that for the given swing model the additional positive couple resulted in the clubhead at impact having a lower speed as the club is brought around too early. Of course, the effect of applying any additional positive couple at the hands will depend on the particular swing model. For example, applying a positive couple at the hands after a delayed release can result not only in increased clubhead speed but also in correct timing.

As has been discussed, Jorgensen (1994) and Miura (2001) found that in order to get good agreement between the double pendulum model and experimental measurements the constraint of a fixed hub had to be removed. Jorgensen considered the effect that different lateral shifts, or different accelerations, of the hub have on the impact speed of the clubhead. Removing the lateral shift altogether from the model resulted in the clubhead speed at impact being reduced by approximately 8.8%. Increasing the lateral shift resulted in the clubhead speed at impact being increased by as much as 17%. Any lateral shift will of course need to be timed with the motion of the double pendulum system in order to achieve these improvements in clubhead speed, as well as maintaining the orientation of the club at impact. Miura found that including the upward shift of the hub, during the final 40 ms before impact, in his swing model resulted in an increase in the clubhead speed at impact of approximately 7%.

2.1.3. Triple-link model. Several researchers have used a triple-link model in their analysis of the golf swing. As with the double pendulum model all three links are taken to be swinging in a single plane. The upper link represents the motion of the golfers shoulder as it rotates about the central hub of the golfer. The middle and lower links correspond to the left arm, for a right-handed golfer, and the golf club, respectively. The couple applied at the hub corresponds to the torque generated by the rotation of the golfers hips and trunk. The couple applied by the upper link on the middle link corresponds to that applied by the golfer in the rotation of the arms about the shoulder joint, while the couple applied on the lower link, the club, is τ_h, as with the double pendulum model.

Turner and Hills (1999) present a triple-link model of the golf swing in which both the backswing and the downswing are considered. Separate sets of constant couples are applied at all the pivot points for both the backswing and the downswing. The switch between the backswing and the downswing couples occurs during the backswing, and in their model the upper link ends up starting in the downswing while the club is still continuing in the backswing. Experimental measurements of the torques exerted by players in static positions allowed Turner and Hills to obtain estimates of the couples that correspond to τ_0, $\tau_{shoulder}$ and τ_h in the model. In the swing model that they present the couples applied by the hub, shoulder and hands were set to constant values of −14, −26.8 and −7.3 Nm for the backswing and 105, 75 and 20 Nm for the downswing. The timing of the simulated swing was found to be sensitive to the relative values of these two sets of couples.

Kaneko and Sato (2000) use a triple-link model along with optimal control theory to determine the time evolution of the couples exerted at the hub, shoulder and hands that corresponded to the minimization of certain criteria. It was found that the couples that correspond to a minimization of the total power expenditure agreed quite well with the couples that they determined from measurements taken from the recorded downswing of a golfer. The optimal couples, under this criterion, for both the hub and the shoulder were found to increase up until approximately 50 ms before impact, after which they both dropped down to approximately zero at impact. Both the hub and shoulder couples reached peak values of approximately 175 N m. Using the minimum power expenditure criterion for their swing model, Kaneko and Sato then considered the expected effect that increasing the club mass, length and impact speed would have on the applied couples.

2.2. Measurements taken on golfers

2.2.1. Kinematic and kinetic measurements. In order to measure the positions of the club and the golfer during the downswing, researchers have taken high-speed films, videos or stroboscopic photographs of the swings of both skilled and unskilled golfers. These measurements have then allowed for the determination of the linear and angular velocities and accelerations of both the club and the golfer. Some of the results of these particular studies will now be considered.

One of the simplifying approximations made in the double pendulum swing model is that the golf club swings in a plane over the duration of the downswing. Vaughan (1979) considered this and measured the variations of the instantaneous plane of the golf club for a particular swing. It was found that there was significant variation in the early stages of the downswing, approximately the first 100 ms, up until the point where the golfer's arms are approximately in the horizontal position. After this point, corresponding to the last 100 ms of the downswing, the plane of the swing was fairly well established. This seems reasonable in that, during the initial stages of the downswing where a constant wrist-cock angle is maintained, the motion of the arms primarily determines the position and motion of the club. After the club is allowed to swing out, the motion of the club will be primarily determined by the centrifugal forces acting on it.

Using measurements taken from the photographed swings of skilled golfers, Milburn (1982) determined the time evolution of the angular velocities and accelerations of both the club and arms and Vaughan (1979) determined the time evolution of the speed of the golfer's hands and the clubhead. Both found that the velocity of the arms or hands reached a maximum approximately halfway through the downswing and then decreased until impact. This agrees with the double pendulum model and the natural transfer of energy from the upper link to the lower link. The velocity of the clubhead in the case of Vaughan's golfer increased steadily right up until impact, while for Milburn's golfer the angular velocity of the club peaked just prior to impact.

Cooper and Mather (1994) also determined the time evolution of the angular velocity of the club in their analysis of the swings of professional, low-handicap and high-handicap golfers. In the case of the professional golfer, the angular velocity of the club was found to peak exactly at ball impact, corresponding to Vaughan's result. For low-handicap golfers the angular velocity curve peaked just before impact, as found with Milburn's golfer. For high-handicap golfers it was found that the angular velocity of the club increased very rapidly during the first half of the downswing but peaked well before impact and then decreased substantially before impact. Mather (2000) found that the arms and shaft of the weaker golfers are almost in line after the arms have turned through 90° and not only is there no further acceleration of the clubhead after this point, but the position of the body and arms may promote a deceleration. In terms of the two-stage swing model these results indicate that the less skilled golfers are releasing and accelerating the club much too soon. A comparison between the resulting evolution of the clubhead speed of a high-handicap amateur and a professional golfer is shown in figure 5.

The early release of the club by less skilled golfers is also shown in the results of Robinson (1994), who measured a series of swing characteristics for both professional and amateur golfers. Of all the characteristics that Robinson measured, the most significant, in terms of correlating with the clubhead speed at impact, was the wrist-cock angle at the point in the downswing where the left forearm is parallel to the ground. At this point in the swing it was

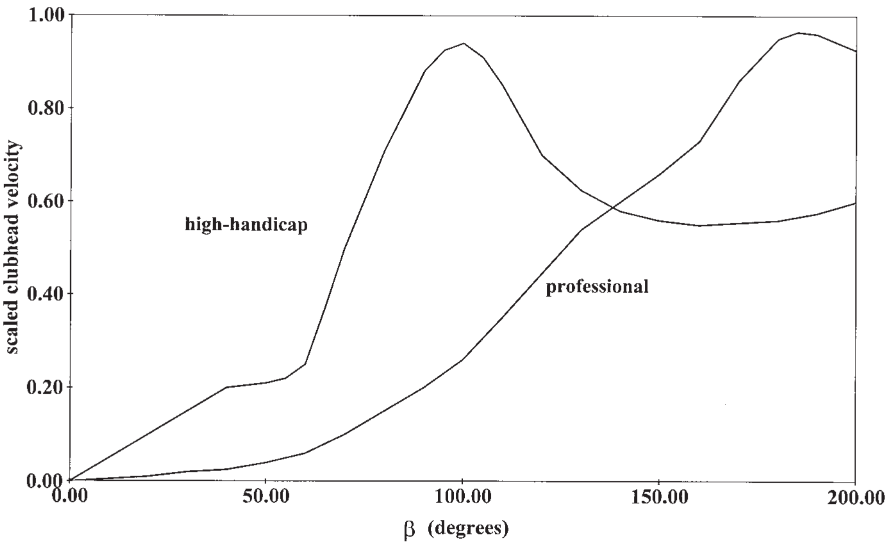

Figure 5. A comparison of the evolution of the clubhead speed for a high-handicap amateur and a professional player. Adapted from figure 1 in Mather (1999). β represents the angle into the swing with the impact occuring at 190°.

found that, for the professional golfers, the average wrist-cock angle was approximately 100° while for the amateurs it was only 77°. This would indicate that the amateurs had, by this point in the downswing, already released the golf club while the professionals were still maintaining their initial wrist-cock angle.

Several researchers have determined, from measured angular velocities and accelerations of the arms and club, the time history of the resultant couple that must have been exerted, in terms of the swing model, at the hub and at the hands. Lampsa (1975), from the analysis of his own swing, found that as the downswing proceeded, the hub couple steadily increased until 100 ms before impact, reaching a peak value of approximately 270 N m. The hub couple then decreased, falling to zero at impact. Kaneko and Sato (2000) found a qualitatively similar result for the calculated couples exerted at the hub and shoulders in their triple-link model of the swing of a particular golfer. In this case, the peak values for both couples were approximately 150 N m, and occurred at 50 ms before impact. These results show that the constant hub couple often used in swing models is, at best, a very rough approximation.

Lampsa (1975), Vaughan (1979), Budney and Bellow (1979), and Kaneko and Sato (2000), all determined the couple that must have been exerted by the hands of their golfers from measurements of the angular acceleration of the club. Although there are significant differences, all these researchers found that the couple exerted by the hands stayed positive until late into the downswing, when it reversed and then stayed negative until impact. This behaviour, with the couple staying negative until impact, is very different from that normally associated with the two-stage double pendulum model.

Neal and Wilson (1985) also determined the couple exerted by the hands of a golfer during the downswing. They also found that, for their golfers, the hands and wrists exerted a positive couple followed by a negative couple. However, the couple applied at the hands and wrists reversed and was positive for the final 30 ms of the downswing, which resulted in a steadily increasing clubhead speed up until impact. The application of a positive couple at the hands late in the downswing agrees with the result of Budney and Bellow (1990) who measured the pressure exerted on the grip by the hands of a professional golfer by fitting a driver with transducers. One of their findings was that the right hand of the right-handed golfer although relatively passive at the start of the downswing, applied a force impulse that peaked approximately 50 ms prior to impact. This force impulse by the bottom hand would result in a positive couple being exerted on the club.

There, thus, appear to be fundamental differences in the ways golfers use their hands prior to impact. If golfers swing out the club too early in the downswing they will need to apply a negative couple at the hands later in the downswing in order to retard the swinging out of the club so as to have it in the proper position at impact. If the release of the club is delayed too much, a positive couple would need to be applied at the golfers hands in order to get the club around in time for impact. A benefit of this action is that applying a positive couple to the club just prior to impact would result in an increased clubhead speed.

Several researchers, using high-speed video, have made measurements of the rotation of the hips about the feet and the rotation of the trunk about the hip during downswings. These rotations will be a major contributor to the hub couple. McTeigue *et al* (1994) found that professional golfers generally rotate their hips first in the downswing followed by the rotation of the torso. They also found that although both the professionals and amateurs whom they considered had their hips and torso rotated back the same amount, approximately 55° and 87° respectively, at the top of the backswing, the amateurs rotated around much slower during the downswing taking an additional 31% more time. Robinson (1994) also measured the angular velocities of the hips and found that the professional's hips, on average, were rotating 28% faster than the amateurs at approximately the mid-point of the downswing.

Watanabe *et al* (1999) used a body-twist motion jacket sensor and measured the angle between the shoulders and hips, or what they referred to as the body-twist angle, during the downswing. They found large differences between a particular low-handicap and very high-handicap golfer. Not only did the low-handicap golfer have 50% more twist than the high-handicap player, he also generated approximately twice the body-twist velocity throughout the downswing. Surprisingly, this only resulted in the low-handicap player having a clubhead speed 8.6% greater. This would imply that the high-handicap golfer compensated with greater shoulder torque, a technique that is often observed with high-handicap golfers.

2.2.2. Force measurements. In addition to the analysis of golf swings based on high-speed photography, researchers have also directly measured the forces exerted by the golfer. These measurements generally fall into two categories, the measurement of ground reaction forces through the use of force plates and measurements of muscle activity through electromyography. Dillman and Lange (1994) have written a review on the biomechanics of the golf swing. This includes the measurements of the ground reaction forces that have been made and electromyography. A brief summary of some of the key findings will be presented here.

From ground reaction force measurements researchers have found that golfers start the downswing with their weight shifted to their back foot. As an example, Koenig *et al* (1994) found that, for the golfers that they measured, approximately 65% of their weight rested on their back foot at the top of the backswing. The golfers then rapidly shifted their weight to their front foot during the downswing. Skilled golfers are found to transfer their weight at a fast rate and reach a peak, in terms of the weight resting on the front foot, near mid-downswing. Less skilled golfers are found to transfer their weight at a much slower rate with peak weight transfer coming later in the downswing. In addition, the actual weight transfer for both the backswing and the downswing is found to be much less for the less skilled player. These results would seem to indicate that the more skilled players generate more power through the use of the large muscles of the legs and hips while the high-handicap player relies more on the swinging of his or her arms. In terms of the swing model, these results also imply that a skilled golfer is applying a greater lateral shift of the hub than are the unskilled players.

Electromyography measurements have shown that the major difference between skilled and unskilled golfers is not so much in the individual forces generated by the muscles but their coordination and consistency. Indeed, Hosea *et al* (1990) found that the myoelectric activity in the backs of the professional golfers that were tested, were much less than that found for the amateurs. In general, muscle activation patterns are found to be very consistent from swing to swing for expert subjects while they are highly variable for high-handicappers. Muscle coordination and timing patterns, therefore, appear to be crucial in obtaining high swing velocities.

3. The physics of the golf ball

Few sports items of such apparent simplicity have undergone more study and analysis than the golf ball. Modern day golf balls come in a variety of different constructions using a wide range of different materials, as manufacturers continually strive to produce a better golf ball. The first topic considered in this section will be the measurements and models of the normal and tangential forces exerted on the various golf ball constructions during impact with the clubhead or with fixed barriers. The normal force will determine the launch speed of the golf ball while the tangential force will determine the amount of spin.

One of the most interesting phenomena in the game of golf is the distance the ball can be driven. The large distances are due to the effect that the dimples on the cover as well as the spin of the golf ball have on the aerodynamics. Measurements of lift and drag coefficients will be presented, including the effect of various dimple patterns.

The final topic that will be considered is the interaction between the golf ball and the turf. Golfers are especially concerned with the amount of bounce and roll for shots that land on the green. Various models of the run of a golf ball will, therefore, be presented as well as the behaviour of a putted golf ball.

3.1. Impact between golf ball and clubhead

3.1.1. Normal forces and the coefficient of restitution. The impact between the clubhead and a golf ball is a very violent event. The normal force acting on the golf ball during impact is large, with values reaching 10 kN, and the apparently solid ball is deformed significantly, with compression along the direction of impact being of the order of 1 cm. A golfer will, in general, want a golf ball launched with maximum possible speed. The measured characteristic of a golf ball that directly corresponds to its maximum possible launch speed is its coefficient of restitution. The coefficient of restitution is defined as the ratio of the normal component of the velocity of the ball, relative to the clubhead, after impact, to its velocity relative to the clubhead before impact, and for a perfectly elastic collision has a value equal to 1.00. A golfer will typically want a golf ball with a high coefficient of restitution.

In general, the greater the deformation of the golf ball the greater the energy loss and the lower the coefficient of restitution. The coefficient of restitution will, therefore, normally decrease with impact speed as a result of the greater deformation of the golf ball. As an example, Chou *et al* (1994) present results showing the dependence of the coefficient of restitution of a one-piece solid golf ball on its impact speed. The coefficient of restitution was found to drop from approximately 0.85, for an impact speed of $20\,\mathrm{m\,s^{-1}}$, down to approximately 0.78, for an impact speed of $45\,\mathrm{m\,s^{-1}}$. Other researchers, such as Ujihashi (1994), have reported data showing a similar dependence of the coefficient of restitution on impact speed.

In addition to the impact speed, the structure of the golf ball will also affect its coefficient of restitution. Historically, golf balls have been classified as being of either two-piece or three-piece construction. A standard two-piece ball will typically have a solid rubber core with a hard ionomer blend cover while a standard three-piece ball will have either a solid rubber or liquid filled core wrapped in a layer or rubber windage and will normally be covered with a softer synthetic balata. The standard two-piece ball has traditionally been known for its durability and its performance in distance while professionals have normally preferred the standard three-piece ball for its ability to ‘hold the green’ with its higher spin rates. Nowadays, all combinations of cores and covers can be found on the market as well as multilayer golf balls, which consist of three or more solid layers of various materials.

As the first step towards a fundamental understanding of how golf ball construction affects its coefficient of restitution, researchers have measured and modelled the normal force exerted on the different golf ball constructions during collisions with fixed barriers. Gobush (1990), Ujihashi (1994), and Johnson and Ekstrom (1999), have all presented measurements of the time history of the normal force for both standard two-piece and three-piece golf balls fired from air cannons onto force transducers mounted to fixed barriers. In the case of Ujihashi, the impact between the ball and the barrier was normal while Gobush, and Johnson and Ekstrom considered oblique impact.

In general, it is found from these normal force measurements that, at a given impact speed, the standard three-piece ball is in contact longer with the barrier than the standard two-piece ball. As an example, Ujihashi (1994) found that the average contact time for three-piece balls was 15% greater than the contact time for two-piece balls, approximately $480\,\mu\mathrm{s}$ to $420\,\mu\mathrm{s}$

respectively. These force transducer measurements agree, in general, with those of Roberts *et al* (2001) who directly measured the electrical contact time between conductive-coated golf balls and various clubheads. Roberts *et al* found that three-piece balls were in contact for approximately 16 μs longer than two-piece balls.

Although the three-piece ball is found to be in contact longer, at normal or low angle impacts the peak normal force and the coefficients of restitutions for both two-piece and three-piece balls are found to be similar. As an example, Gobush (1990) found that, for a 20° oblique impact, with an impact speed of 29 m s^{-1}, the peak normal force acting on a two-piece ball was 8.49 kN compared to 8.71 kN for a three-piece ball. By integrating the normal time histories Gobush also found that both the two-piece and three-piece balls had a coefficient of restitution of 0.78 at this impact angle. The shape of the time histories of the normal force exerted on the two-piece and three-piece balls were, however, quite different. The three-piece ball exhibited distinctive shoulders about the central maximum. Such shoulders are normally characteristic of balls with a hollow centre.

At higher impact angles both Gobush (1990), and Johnson and Ekstrom (1999), found greater differences between two-piece and three-piece balls. For the 40° oblique impact that Gobush considered and the 45° oblique impact of Johnson and Ekstrom, the peak normal force is approximately 10% greater for the two-piece ball. Gobush calculated the coefficient of restitution for the two ball types and found that the value for the two-piece ball, 0.78, was significantly higher as compared to the three-piece ball whose value was 0.68. The deformation of the two-piece ball was also significantly less, by approximately 8%, as compared with the three-piece ball. Two-piece balls are, therefore, found to behave more stiffly, with a corresponding higher coefficient of restitution, than a three-piece ball at the greater impact angles. Golfers, in general, do find that standard two-piece balls fly farther for the higher lofted clubs.

These results are in line with the results of Mather and Immohr (1996) who carried out compression tests on both two-piece and three-piece golf balls. These results were then used to calculate the dependence of the elastic modulus on the amount of deflection. The two-piece ball was found to have a much greater elastic modulus at smaller loads as compared to the three-piece ball. However, as loads approached 11 kN, the elastic modulus for both ball types approached a value of 90 MPa. Therefore, for the higher loads experienced at the lower impact angles the two ball types will have approximately the same stiffness, while for the lower loads experienced at the higher impact angles the two-piece ball would be expected to behave more stiffly than the three-piece ball.

Chou *et al* (1994) and Taveres *et al* (1999a) used finite element analysis to simulate the normal impact of golf balls. Chou *et al* modelled the golf ball as a disc and found that the simulated impact provided a good fit to the experimental time histories of the impact forces and the dependence of the coefficient of restitution on normal impact velocities as given by Gobush (1990). Chou *et al* also used the model to show how the coefficient of restitution depends on both the PGA compression number, which is related to the Young's modulus of the ball, and on the viscosity of the golf ball. As expected, the coefficient of restitution increases with the PGA number, or the stiffness of the ball, and decreases with increasing viscosity. Taveres *et al* measured and simulated the normal force time history for both two-piece and multilayered golf balls. Good agreement was obtained between the measured and the simulated results and the multilayered normal force time history was found to be similar to that of a standard two-piece golf ball.

Several researchers have presented mechanical models of the normal impact of a golf ball against a barrier. In these models the deformation arising from impact is equated to the motion of a lumped mass in combination with nonlinear elastic and dissipative elements. The simplest example is the two-parameter model presented by Cochran (1999) in which the elastic element

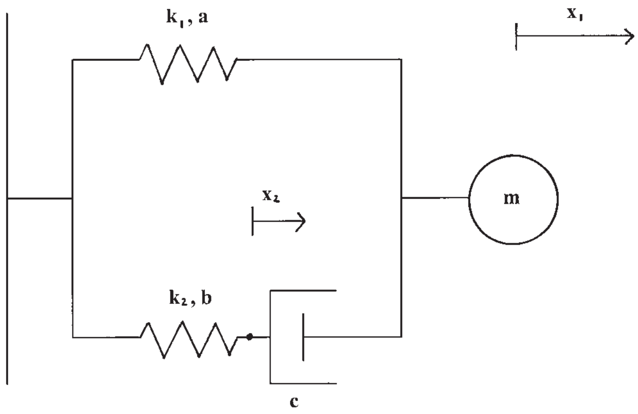

Figure 6. The five-parameter model of the golf ball. Adapted from figure 1 in Johnson and Lieberman (1994a).

behaves as a Hertzian spring. The force exerted by a Hertzian spring is proportional to $x|x|^{1/2}$, where x is the amount of deformation. This is the theoretical behaviour of a solid, uniform sphere, undergoing small deformations against a plane surface. The reason for this nonlinear behaviour is that as the ball compresses against the barrier the area in contact increases resulting in a greater stiffness. In Cochran's model the elastic element is in parallel to a dissipative element, which by analogy with the elastic force, is taken to be proportional to $\dot{x}|x|^{1/2}$. This fairly simple two-parameter model results in the coefficient of restitution decreasing with impact speed, in agreement with the experimental results of Chou *et al* (1994) and Ujihashi (1994).

Increasing the number of elements and parameters in the mechanical model will, of course, allow for better agreement with the measured normal time histories and their dependence on impact velocity. Johnson and Lieberman (1994a, 1996) have presented a five-parameter mechanical model of the golf ball for normal impact with a fixed barrier. The model consists of a lumped mass attached to a series and parallel combination of two nonlinear elastic elements and one dissipative element. This mechanical model is shown in figure 6. In this model the forces exerted by the elastic elements are taken to be proportional to a power of the strain. Two parameters are, therefore, required for each elastic element, one for the exponent and one for the proportionality constant, while one parameter is required for the dissipative element, whose force is taken to be proportional to the ball speed. The mathematical equations of motion for this model are

$$m\ddot{x}_1 = -k_1 x_1^a - c(\dot{x}_1 - \dot{x}_2) \tag{3a}$$

and

$$c(\dot{x}_1 - \dot{x}_2) = k_2 x_2^b. \tag{3b}$$

Values for k_1 and a were estimated from compression measurements while the other three parameters were determined by obtaining the best fit to normal force time histories. This five-parameter model provides a good fit to the normal time histories of both two-piece and three-piece balls. In the case of two-piece balls, Johnson and Lieberman (1996) further found that very good agreement could still be had if the five-parameter model was reduced to three parameters, by setting the two exponents of the spring forces to the Hertzian value of $\frac{3}{2}$.

3.1.2. Tangential forces and spin. Although differences do exist with normal force time histories and coefficients of restitution, the most distinguishing feature between the different ball constructions is the amount of spin that they acquire during the impact with the clubhead. The amount of backspin affects the aerodynamics of the launched ball and in general there will be, in terms of maximizing the drive distance, an optimal amount of backspin. The amount of backspin will also affect the ability of a golfer ‘to hold the green’, which refers to limiting the bounce and roll of the ball on landing for short iron shots to the green. The amount of sidespin imparted to the ball will affect the ability of the good golfer to draw and fade the ball while affecting the amount of hook and slice for the weaker player. The tangential or frictional forces exerted on a golf ball during impact will determine the amount of spin acquired by it.

Gobush (1990), and Johnson and Ekstrom (1999), all measured the time histories of the tangential force acting on the surfaces of both standard two-piece and three-piece golf balls fired obliquely onto barriers. Johnson and Ekstrom found that for 45° impacts, at impact speeds of 42.7 $\mathrm{m\,s^{-1}}$, the tangential force exerted on three-piece balls during the collision reached significantly greater values than for the two-piece balls, approximately 2.5 kN and 1.5 kN respectively. Gobush found a similar result for the 40° impact at an impact speed of 29 $\mathrm{m\,s^{-1}}$ with a peak tangential force for the three-piece ball of 1.29 kN as compared to 1.08 kN for the two-piece ball. For both these relatively high impact angles, the tangential force was found to oppose the initial motion of the ball throughout the collision. However, at a lower impact angle of 20°, Gobush found that the time history of the tangential force changed significantly. For both the two-piece and the three-piece ball constructions, the tangential force reversed direction approximately 100 μs before the ball left the barrier. It was during this stage that the differences between the wound three-piece ball and the two-piece ball became evident. Gobush found that the magnitude of the tangential force, after reversing direction, reached values that were approximately 40% greater for the two-piece ball than for the three-piece ball.

Using the measured tangential forces, Gobush calculated the torque, which is the product of the tangential force and the instantaneous ball radius, over the duration of the collision for both the 20° and the 40° impacts. Given the torque, the time evolutions of the spin of the two golf ball constructions were then determined. For the 40° impact, the greater measured tangential force acting on the three-piece ball resulted in its calculated spin at rebound being significantly greater, 123 rps compared to 107 rps, than for the two-piece ball. This result agreed well with experimentally measured spin rates, found using stroboscopic photography. For the 20° impact, the calculated spin rate for both ball types were found to be approximately the same, 107 rps, at the point where the tangential force reversed direction. After this point the calculated spin rates of both balls decreased until rebound. The lower tangential force during this period resulted in the calculated spin rate at rebound for the three-piece ball, 72.9 rps, being greater than the corresponding value for the two-piece ball, 63.1 rps. These values were in reasonable agreement with the measured rebound spin rates, again found using stroboscopic photography.

Hocknell *et al* (1999) used non-contacting transducers, laser Doppler vibrometers, to measure directly the time history of the spin of a two-piece golf ball during the collision with the clubhead of a driver that had a loft of 10.5°. They found that the spin of the ball increased rapidly for the first 100 μs and then slowly but steadily increased over the remainder of the impact reaching a value of 45.8 rps at launch. These experimental results agreed with a simulation using finite element models of the clubhead and the golf ball. Unlike Gobush’s result for a 20° impact angle, the spin of the golf ball did not decrease during the final stages of the rebound. It may be that Gobush’s calculations, which are based on tangential force measurements, relate more to the average spin of the whole ball while the experimental measurements of Hocknell *et al* relate to the motion of the cover. One of the difficulties in calculating or measuring the time history of the spin of an impacting golf ball is that the golf ball is greatly deformed during the collision. The spin of a golf ball during the collision is, therefore, not clearly defined.

As a first step in modelling the acquisition of spin by a golf ball, the ball can be treated as a rigid body. On impact the golf ball will start to slide along the face of the barrier. However, friction will retard this motion and the ball will be put into a state of rolling. For relatively high impact angles the golf ball will be in a combined state of sliding and rolling throughout the impact, while for low impact angles the ball would be expected to be in a state of pure rolling at launch. Although this simple model is adequate to explain the general dependence of the acquired spin rates on clubhead loft and impact speed it does not adequately explain details of the tangential force measurements and the spin rates found with the various golf ball constructions.

In order to understand the effect that ball construction has on spin rate, Johnson and Lieberman (1994b) constructed a mechanical model of the oblique impact of a golf ball and a barrier that partially takes into account the non-rigid nature of the golf ball. In this model a torsional component, consisting of a torsion spring and a dissipative element, is added to their five-parameter model for normal impact. The golf ball is modelled as having an outer shell concentric with a rigid inner core. The relative angular motion of the shell and core is governed by the values of the torsional elements. At impact the transverse motion of the shell along the barrier is retarded by friction. However, with this model, the shell and the core undergo separate angular accelerations and the core will initially try to drag the shell after it. Johnson and Lieberman simulated the impact between this ball model and a barrier and found that at the relatively high impact angle of 55° the shell was sliding throughout the impact, with the frictional force exerted by the barrier opposing the initial motion of the golf ball for the full contact time. However, for impact angles of 45° and lower, they found that the shell was put into a state of rolling at some point during the collision. The ball remained in this rolling state until late into the rebound stage when the frictional force decreased to the point where it could no longer sustain rolling, and the ball then began to slide. For the parameters used in this simulation the core was oscillating relative to the shell during impact and the simulated tangential force acting on the shell was found to reverse direction during the collision. The simulated time histories of the tangential force between the golf ball and the barrier agree, in their general behaviour, with the experimental results of Gobush (1990).

Johnson and Lieberman (1994b) used their model to compute spin rates for various golf ball impact speeds and angles. For their particular parameters and at an impact speed of $30.5\,\mathrm{m\,s^{-1}}$, the backspin of the golf ball was found to increase from 32.5 rps, at an impact angle of 15°, to 127 rps, for an impact angle of 45°. However, at the even greater impact angle of 55°, the backspin calculated was only 116 rps.

Measurements by Gobush (1990) and Lieberman (1990) show that a standard three-piece golf ball acquires more spin during impact with a barrier, than a standard two-piece ball. For example, Lieberman found that at an impact speed of $36.6\,\mathrm{m\,s^{-1}}$ the measured backspin of a three-piece ball was 70 rps, 103 rps, 150 rps and 178 rps for impact angles of 15°, 25°, 35° and 45° respectively. The corresponding values for a two-piece ball were 55, 91, 131 and 171 rps for the same impact angles.

The reason for the differences between the spin acquired by the two ball constructions is due both to the hardness of the cover and to differences between the internal constructions. A standard three-piece ball has a softer synthetic balata cover, which has a modulus of elasticity several times less than the hard ionomer cover of a standard two-piece ball. A softer cover would result in a greater coefficient of friction. At the higher impact angles, where the deformation of the golf ball is less and it may not reach a state of rolling, the greater frictional force will be the dominant factor in determining the spin rate.

Gobush (1996a) measured the coefficient of friction between both soft covered three-piece and hard covered two-piece balls and a club insert that was mounted on a force transducer. The coefficients of friction were determined from both the ratio of the measured transverse and normal forces and from calculations using measurements of the velocity components of the balls before and after impact. For a relatively high angle of incidence of 70° and an incoming tangential speed of 12.8 m s^{-1}, both methods gave a value of approximately 0.38 for the three-piece ball and 0.16 for the two-piece ball. The rebound spin rates were found to be 66.2 rps for the three-piece ball and 25.5 rps for the two-piece ball. For a greater incoming tangential speed, 26.8 m s^{-1} for the three-piece balls and 25.9 m s^{-1} for the two-piece balls, the average coefficient of friction decreased to approximately 0.29 and 0.075 respectively.

Sullivan and Melvin (1994) specifically considered the effect that cover hardness, measured on the Shore D scale, had on the acquired backspin of a 9-iron shot that had a clubhead speed of 32.0 m s^{-1}. Measured spin rates dropped from approximately 175 rps for a relatively soft cover, approximately 50 on the hardness scale, down to approximately 110 rps for a relatively hard cover, approximately 67 on the hardness scale. They also directly compared two-piece and three-piece balls with different covers. For two-piece balls with hard ionomer covers the amount of backspin acquired was approximately 140 rps, compared to 167 rps for a two-piece ball with a synthetic balata cover. Similar results are found with the three-piece balls with hard and soft covers.

At lower impact angles, where the ball starts rolling at some point during the impact, the acquisition of spin is more complex. In these cases, the final spin acquired by a golf ball will depend not only on the friction between the cover and the barrier, but also on the nature of the internal wind-up of the deformed ball, as well as when and how much the golf ball slides during the final stages of the rebound. Gobush (1996b) presents a model, similar to that of Johnson and Lieberman (1994b), where the golf ball is taken to consist of a central core surrounded by nine concentric layers. In the model each layer is connected to the neighbouring layers by a torsion spring. The three-piece ball is modelled as having the stiffness of the springs increasing towards the core while the two-piece has the stiffness of the springs decreasing towards the core. The spin acquired by the two models for a variety of oblique impacts and different coefficients of sliding friction were computed. The three-piece ball model was found to acquire more spin in almost every circumstance.

Sullivan and Melvin (1994) looked at the dependence of the acquired spin of a golf ball on its compressibility. In general, they found that a golf ball with a hard core acquires more spin than a soft-core ball that has the same cover. The reason given is that a hard core serves to compress the cover of the ball against the clubface to a much greater degree resulting in greater friction.

Tavares *et al* (1999b) used finite element analysis to investigate spin acquired by two-piece and multilayered golf balls with different cover hardnesses. They simulated impact for the full range of clubs, from the driver to the pitch shot, for clubhead velocities typical of professional golfers. They found that the two-piece ball with the softest cover had the greatest spin for all shots. They also found that a multilayered golf ball, with a hard mantle layered between the core and a soft cover, could match the high spin of the soft covered two-piece ball obtained with the high lofted irons but had significantly less spin for the driver. The simulation showed that the mantle layer absorbed some of the impact forces that can generate excess spin. They found good agreement between their results, using finite element analysis, and the measured launch conditions of different ball types struck by professional golfers. The benefit of such a golf ball, which has high spin rates for the high-lofted irons and relatively low spin rates for the driver, is that it would allow the good golfer control around the greens without loss in the drive distance.

3.2. Golf ball aerodynamics

After the golf ball leaves the clubhead its motion is governed by the force of gravity and the aerodynamic forces that are exerted on it by the air. This section will look at the various aerodynamic measurements that have been carried out on golf balls. Figure 7(*a*) shows the basic features of the air flow pattern around a golf ball. The separation point is the position where the boundary layer, which is the thin air layer dragged by the surface of the ball, separates from the surface. In the figure shown, the boundary layer separates just downstream of the sphere's mid-section. The wake that is created is a region of relatively low pressure and the resulting pressure difference between the forward and rearward regions results in a pressure drag on the body. The greater the size of the wake the greater the drag will be. As the air flow velocity increases, the separation point moves forward towards the ball's mid-section, and at high enough velocity into the forward half of the golf ball. During this stage the drag increases and is proportional to the square of the ball's speed. However, if the ball speed continues to increase, and the critical Reynolds number is reached, the boundary layer in the forward portion of the sphere becomes turbulent. When this happens the point of separation moves back downstream. The wake is thereby dramatically reduced along with the resulting pressure drag, which drops to about half of its pre-critical value.

For a smooth sphere, Achenbach (1972) found that the critical Reynolds number is about 3×10^5, which corresponds to a ball the size of a golf ball travelling at around $110\,\mathrm{m\,s^{-1}}$ through the air. As the launch velocity of a golf ball off a driver is far below this, only about $80\,\mathrm{m\,s^{-1}}$ even for a 300 yard drive, the critical Reynolds number would not be reached if the surface of the golf ball were smooth. However, any roughness of the sphere, such as dimples, will accelerate the onset of turbulence in the boundary layer and thereby reduce the critical Reynolds number. Aoki *et al* (1999) determined the separation points for both smooth and

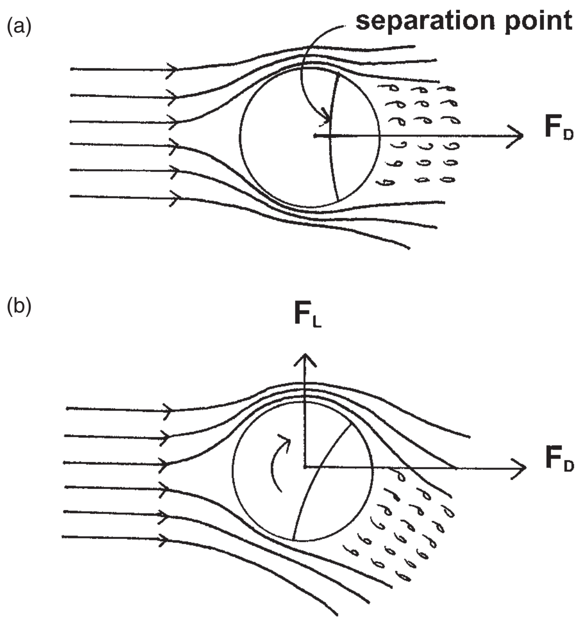

Figure 7. The airflow and resulting forces acting on a golf ball moving to the left without backspin (*a*) and with backspin (*b*).

dimpled golf balls using oil film and spark tracing methods. They found that in the range of flying speeds of 6–42 m s^{-1} the flow around the smooth sphere remained laminar, whereas the boundary layer for the dimpled ball became turbulent at about 21 m s^{-1} corresponding to a critical Reynolds number of 6×10^4. The corresponding decrease in the size of the wake is very evident in the photographs they present. The effect of dimples on a golf ball is further highlighted by Fox and McDonald (1992) who obtained samples of golf balls without dimples. The average drive distance of these balls in tests was only 125 yards compared to 215 yards for dimpled golf balls.

A spinning golf ball travelling through the air will have, in addition to drag, a force perpendicular to the ball's velocity, commonly referred to as lift. Figure 7(*b*) shows the airflow pattern around a spinning golf ball, in this case equivalent to a golf ball travelling to the left with backspin. As is shown in the figure, the boundary layer is dragged around by the spinning ball and the separation point is delayed on the upper surface while occurring earlier on the lower surface. The photographs of Aoki *et al* (1999) clearly show the separation point shifting downstream at the top of the spinning ball and shifting upstream at the bottom. The resulting differences in the air flow speeds over the top and bottom surfaces will result in a net upward force, or lift, being exerted on the golf ball.

It is conventional when measuring drag and lift forces on a golf ball to present results in terms of drag and lift coefficients. The drag coefficient, C_D, is defined as

$$C_D = \frac{F_D}{(1/2)\rho v^2 A}, \tag{4a}$$

where F_D is the drag force, ρ is the density of the fluid, v is the speed of the object through the fluid, and A is the cross-sectional area. Similarly the lift coefficient, C_L, is defined as

$$C_L = \frac{F_L}{(1/2)\rho v^2 A}, \tag{4b}$$

where F_L is the lift force.

Smits and Smith (1994) determined values for the drag and lift coefficients of a golf ball in wind tunnel tests. The range of Reynolds numbers and spin rates used covered the flight conditions experienced by a golf ball for the full range of clubs. The lift coefficients were found to increase with spin rate, as would be expected, and to be approximately independent of Reynolds numbers. For Reynolds numbers greater than 5.0×10^4 the drag coefficient is also found to increase with spin rate. However, unlike the lift coefficients, the drag coefficients increase with Reynolds numbers up to 2.0×10^5. Smits and Smith provide empirical equations for drag and lift coefficients, as well as the spin decay rate, for Reynolds numbers and spin rates applicable to driver shots.

Researchers have also looked at the effect that different dimple patterns and shapes have on the lift and drag acting on a golf ball, as well as the resulting trajectories and drive distances. Stanczak *et al* (1999) made extensive hot wire velocity measurements in the turbulent wake of three golf balls with markedly different dimple patterns. They found significant differences in the size of the wakes for the three golf balls as one moved downstream. This correlated with measured drive differences with the ball leaving the widest wake having drive distances that were approximately 8% less than the ball with the smallest wake.

Aoyama (1994) determined drag and lift coefficients in wind tunnel tests for a wide range of velocities and spin for both older and newer golf balls. Over the majority of the range of Reynolds numbers and spin rates tested it was found that a modern ball, with a 384 dimple icosahedron pattern, had drag and lift coefficients of the order of 5% lower than older balls with a 336 octahedron pattern. Aoyama used the data to computer simulate the trajectories and compared the result with driving tests. For relatively low launch angles and spins, 7° and 43.3 rps respectively, it was found in both the simulation and the driving tests, that the older ball would give approximately a 5 yard advantage for a professional golfer's drive. For relatively high launch angles and spins, 10° and 60.0 rps, just the opposite was found with the modern ball giving a 10 yard advantage. A further advantage of the modern ball is that it will slice less than the older ones. Ayoma estimates that professional players would reduce slices by 2–20% when using the newer golf balls.

Bearman and Harvey (1976) used a wind tunnel to measure lift and drag coefficients for both hexagonally-dimpled and conventional round-dimpled balls over speeds ranging from about 14 to 90 m s^{-1} and for spin rates up to about 104 rps. They found that the golf balls with the hexagonally shaped dimples had higher lift coefficients and slightly lower drag coefficients than the conventionally dimpled balls. They calculated trajectories using their results and compared the calculated range with measurements taken with a driving machine. They found good agreement with the hexagonally dimpled balls but the calculated range of the round-dimpled balls was slightly less than the experimental values. The driving machine results show that a hexagonally dimpled ball travels approximately 6 yards farther than a round-dimpled ball under normal driving conditions.

Tavares *et al* (1999) measured the spin rate decay of golf balls with significantly different dimple patterns. They found, at the high spin rates found with the high-lofted irons, that the dimple pattern can have a significant influence on the rate of spin decay with differences of the order of 30%. The effect that a golf ball's moment of inertia has on its ability to retain spin was also considered. As expected, the greater the moment of inertia the greater the retainment of spin. This is important to golfers who want to be able to 'hold the green'.

MacDonald and Hanzely (1991) modelled trajectories of golf balls using the lift and drag coefficients determined by Bearman and Harvey (1976). They determined that the carry of a golf ball depends approximately linearly on launch speed. This agrees with the empirical results of both Williams (1959), and Cochran and Stobbs (1968). They also considered the dependence of the carry on launch angle and found that, for a fixed launch speed, maximum carry would occur at approximately 23°, a good deal smaller than the 45° found for a projectile when aerodynamic forces are neglected.

Erlichson (1983) along with McPhee and Andrews (1988) modelled the drag and lift forces as being proportional to the speed of the golf ball. This is equivalent to having the drag and lift coefficients being inversely proportional to the speed of the golf ball. Using linear models for drag and lift allowed McPhee and Andrews to obtain an analytical solution for the equations of motion. They show that although this model of aerodynamic forces is in general disagreement with wind tunnel measurements it is adequate for the range of initial conditions produced by a driver. Using their model, McPhee and Andrews considered the effect of sidespin and crosswinds on the golf ball's trajectory. One of the results that they found is that a ball hit into a crosswind, and with sidespin, can fly slightly further, by approximately 1%, than the same ball driven under conditions of no wind.

Although backspin and the accompanying lift is certainly one reason for the great carry distances golfers are able to achieve on their drives, too much backspin can actually reduce drive distance. This is due to the higher trajectory, resulting from the increased aerodynamic lift, as well as the slight increase in the aerodynamic drag. There is, therefore, an optimum amount of backspin for a given launch speed and launch angle. Werner and Greig (2000) calculated the dependence of drive distance on spin rate for a fixed launch speed of 63.9 m s^{-1} and a fixed clubhead loft of 10°. They found that for these launch conditions, and for given lift and drag coefficients, the optimum backspin of the golf ball is approximately 57 rps. Bearman and Harvey (1976) , using their measured lift and drag coefficients, found that for a launch angle of 10° and a launch speed of $58\,\mathrm{m\,s^{-1}}$, the maximum range is obtained with a backspin of approximately 60 rps.

3.3. Interaction between golf ball and turf

3.3.1. The run of a golf ball. The run of a golf ball consists of the bounce phase and the subsequent rolling after landing. In the case of a drive, golfers will typically want a long run, while for shots onto the green golfers will want to limit the run of the golf ball.

Several researchers have presented models of golf balls bouncing on turf. In the case of Daish (1972), the general behaviour of a bouncing ball on a rigid surface is modelled, with the golf ball used as an example. Daish considers two specific cases. First, is the case where the ball slides over the surface throughout the impact, which, for the typical impact of a golf ball on turf, will result in the golf ball retaining some of its backspin as it bounces from the surface. In the second case, the frictional force between the ball and the surface is great enough to check the backspin and to have the ball bouncing out of the impact with top spin. Daish determined that the minimum value of the coefficient of kinetic friction, $\mu_{\min}$, between the ball and the surface, required to check the backspin, is given by

$$\mu_{\min} = \frac{2(v_{ix} + r\omega_i)}{7(1+e)v_{iy}}, \tag{5}$$

where v_{ix} and v_{iy} are the impact velocity components, r is the radius of the ball, ω_i is the backspin of the impacting ball, and e is the coefficient of restitution between the ball and the surface. Daish used this model to compute the runs for several different golf shots. For example, he found that for a given wedge shot, where the golf ball had 191 rps of backspin at impact, the ball slid throughout the first bounce and retained enough backspin to check its motion completely on the second bounce.

Daish used a value of 0.5 for e, the coefficient of restitution between the golf ball and the turf, in his calculations. In general, however, this value would depend on the impact speed. Penner (2001a) measured the dependence of the coefficient of restitution for normal impact on the impact speed of the golf ball and found that although the value of e was approximately 0.5 at low impact speeds, less than $1\,\mathrm{m\,s^{-1}}$, for higher impact speeds the value of e decreased, approaching a value of 0.12 as impact speeds approached $20\,\mathrm{m\,s^{-1}}$.

Daish's rigid surface model also leads to several discrepancies with the real behaviour of golf balls bouncing off a compliant turf. Both rebound height and bounce distance are found to differ. For a typical golf shot, a golf ball will create an impact crater in the turf with a depth of the order of 1 cm. Penner (2002b) adapted Daish's model to account for the compliant nature of the turf by treating the effect of the golf ball rebounding from the impact crater as being equivalent to a golf ball bouncing off a rigid but sloped surface. The slope of the equivalent rigid surface was taken to increase linearly with both the impact speed and the angle of incidence of the golf ball. In Penner's model, the golf ball is taken to continue bouncing until the bounce height drops below 5 mm, after which the golf ball is taken to roll until friction brings it to rest. Using this model, it was found that the dependence of the run of a golf ball on the launch speed in the case of drives agreed very well with the empirical results presented by Williams (1959), and Cochran and Stobbs (1968).

Penner also considered the types of runs that would be expected for golf shots using high-lofted clubs. By modelling the run of a typical 9-iron shot, it was found that a golf ball could impact on the turf with enough backspin to cause it to bounce backwards. The total amount that the ball runs backwards, in this case, increases with the amount of backspin that the ball has at impact. In addition, in the case of firm greens, where the coefficient of friction is less than the critical value, $\mu_{\min}$, it is found that golf balls with sufficient amounts of backspin will initially bounce forward before bouncing and rolling backwards. This behaviour is often seen with the golf shots of professional golfers who can impart sufficient backspin to the golf ball. Figure 8 shows examples of the types of runs for given 9-iron shots on a firm green calculated using Penner's model.

Haake (1991, 1994) modelled both the normal and oblique impact of golf balls on compliant turf. For normal impact the turf is modelled as consisting of two layers, with the first layer having an elastic as well as a dissipative component, while the second layer is treated as having only a dissipative component. In the top layer the elastic force is taken to be proportional to the displacement of the ball, while the dissipative forces, due to each layer, are taken to be proportional to the speed of the golf ball as well as the area of the ball in contact with each layer. Haake relates the values of the spring and damping constants to the thickness of the top layer as well as to the soil composition and moisture content of the turf. In the case of oblique impact, the horizontal forces exerted by the turf were formulated in the same manner as the vertical forces with the top layer modelled as being composed of a spring and a damper in parallel and the bottom layer taken to be composed of a single damper.

Haake used the model to determine the dependence of rebound speed and impact depth on the impact speed in the case of normal impact. These modelled results were found to agree, in general, with experimental measurements taken of golf balls dropped or projected onto greens. The model was also used to predict rebound velocities and spins for various oblique impacts. Haake compared these results to measurements of oblique impacts using stroboscopic photography. The general modelled behaviour of a rebounding golf ball agreed reasonably well with the experimental results. It was found that an increase in impact speed caused the ball to rebound at a greater speed and at a steeper angle while an increase in the backspin would cause the ball to rebound at a slower speed and at a steeper angle.

This model of oblique impact is used by Haake to simulate 5-iron and 9-iron shots on both firm and soft greens. It is assumed in this run model that after the first two forward bounces, the golf ball rolls in the direction of the spin. As an example, the model shows that for a 9-iron shot, with the golf ball impacting on a firm green with 148 rps of backspin, the golf

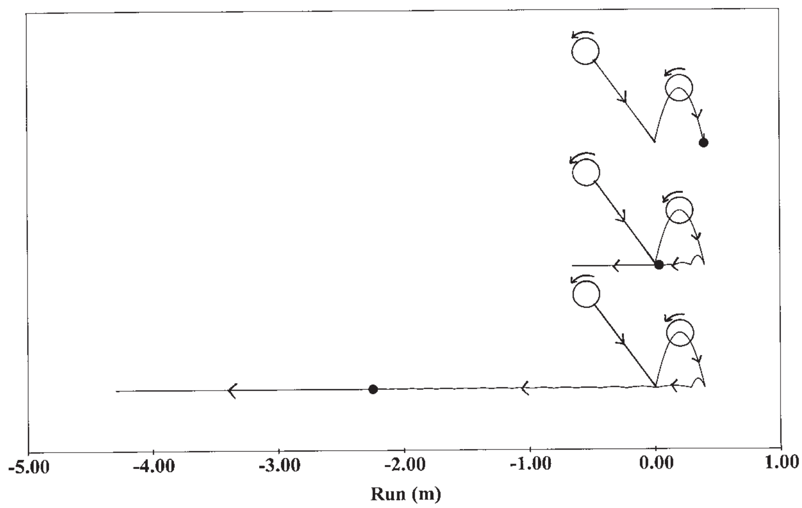

Figure 8. Examples of the run of a golf ball for a 9-iron shot to a firm green. Adapted from figure 8 in Penner (2002b). The impacting golf ball has a backspin of 129 rps (top figure), 159 rps (middle figure), and 191 rps (bottom figure).

ball will, after the first two forward bounces, roll backwards approximately 7 m. In general, Haake found that on a softer green the amount of backspin the golf ball has on impact has less influence on its final position than the equivalent shot on a firm green. The reason being that for the soft green most, if not all, of the backspin is lost during the first bounce.

3.3.2. Putting. Putting is the most common shot in the game of golf. Several researchers have considered the motion of a putted golf ball as well as the interaction between the golf ball and the hole. The initial motion of a putted golf ball is found to be dependent on the manner in which the golfer strikes the ball. Daish (1972) found, from high-speed films, that most golfers putt the ball 'on the up'. For these cases, the golf ball is projected into the air and will, therefore, make a series of bounces before it begins rolling along the green. For putts that are not projected upwards, the golf ball will initially skid along the turf. Both Daish, and Cochran and Stobbs (1968), state that the golf ball will be in a state of pure rolling after the ball has travelled approximately 20% of the total length of the putt.

Lorensen and Yamrom (1992), Alessandrini (1995) and Penner (2002a) have presented models of the motion of a putted ball over a green. The primary difference between the models is the way the frictional force acting on the golf ball is handled. Lorensen and Yamrom use two different constant coefficients of friction, one to model an initial sliding phase, and the other to model the rolling phase. This model was used as the basis for visualizing trajectories, using computer graphics, of golf balls travelling over piecewise-planar models of real greens. Alessandrini treats a putt as a two-point boundary value problem and determines the initial conditions required that would allow the trajectory to terminate at the hole with zero velocity. In this model the frictional force is kept constant over the total length of the putt. Penner (2002a) treats the golf ball as being in a state of rolling throughout the length of the putt. In this model the retarding force acting on the golf ball is taken to be that which constrains the ball to roll. Using this model, Penner determined the trajectories of golf balls putted on flat, uphill, downhill and sideways-sloped greens.

Several researchers have looked at how the golf ball interacts with the hole and more specifically the conditions required for the golf ball to fall into the hole. Mahoney (1982) gives an analysis of a ball interacting with a hole and, surprisingly, finds that the probability of success of an aggressive putt, where the golfer is not concerned with the consequences of missing, is heightened when the attempt is downhill and the green is fast. Holmes (1991) presents the most thorough analysis of the interaction between a golf ball and a hole and determines the overall range of allowable impact speeds and impact parameters that will result in a successful putt. All the possible ways that a ball can be captured by the hole were considered. In the case of an online putt it was found that the maximum speed that a golf ball can have and still be captured is $1.63\ \mathrm{m\,s^{-1}}$. For off-centre impacts the allowed ball speed is less.

Penner (2002a) determined the launch conditions required for a successful putt by determining which golf ball trajectories along the green would lead to allowable impact speeds and parameters as determined by Holmes' ball capture model. As with Mahoney, it was found that the probability of making a downhill putt can be significantly greater than the probability of making an equally distant uphill putt. The reason for this is that for a downhill putt an off-line trajectory will tend to converge back towards the target line, while the reverse occurs for uphill putts. This somewhat surprising result must be tempered by the consequences of a missed putt, as a missed downhill putt will, in general, stop much further from the hole than an equivalent uphill putt. For most golfers this would result in a preference for uphill putts. Penner also determined the range of allowable launch speeds and angles for putts across the slope of a green. It was found that taking a slower uphill path, as opposed to a faster more direct path, would increase the probability of success.

Werner and Greig (2000) did a detailed analysis of several aspects of putting. They looked at hit patterns on a putter and the stop patterns around the hole for golfers of various handicaps. They used these results, along with a model of the putt, to determine what is the ideal distance beyond the hole that a golfer should be aiming for. For example, for a ten-foot putt on a flat green they found that a golfer should strike the golf ball so it would stop a distance of approximately 20–40 cm, depending on the golfers handicap and the green speed, past the position of the hole.

Lemons *et al* (1999) looked at the effects of ball construction on both putting distance and break amounts. They found that the cover hardness played the most important role in the distance the ball rolls. For example, using a putting robot, they found that for the same putter head speed, a hard covered two-piece ball rolled approximately 3–5% farther than either a soft covered three-piece or soft covered two-piece ball. It was also found that the ball construction played no part in the break of the putt.

Hubbard and Alaways (1999) considered several aspects of the interaction between a golf ball and a green. They measured surface viscoelastic properties of the turf by experimentally dropping balls onto a green and measuring their resulting accelerations and positions. Peak accelerations of about $50g$ were measured and even from small drop heights (2 cm) several bounces were observed. They also measured the variation of the coefficient of rolling friction over the length of a putt. They found that the rolling friction increased by about 10% over the course of a 4.3 m putt. This would indicate that the coefficient of rolling friction increases with decreasing velocity.

4. The physics of the golf club

Few, if any, sports require such a range of equipment as does the game of golf. A professional player is allowed to carry up to fourteen golf clubs. These clubs are generally divided into three categories, woods, irons and the putter. For the woods, the modern club consists of a bulbous hollow clubhead, typically constructed out of steel or titanium, attached to a shaft, typically constructed out of steel or graphite. These clubs are designed to hit the ball long. The irons and the wedges have basically solid, flat, trapezoidal shaped steel clubheads and are designed to allow the golfer to control the distance of the shorter shots through the variation in their lengths and lofts. The putter has a clubhead, which itself can come in a variety of shapes, that is designed to give a golfer the maximum control while putting.

In the past the design of golf clubs was as much an art as it was a result of any scientific insight. Over the last several decades things have changed considerably. Nowadays, manufacturers are basing changes of club design more and more on scientific principles and measurements. This section will look at some of the research that has gone into understanding the behaviour of the golf club. It will be grouped under three headings: the shaft, the clubhead and club design.

4.1. The shaft

4.1.1. Effect of length and mass. Researchers have considered the effect that both the length and mass of a shaft have on the generated clubhead speed. It may be expected that the longer the shaft, the greater the generated clubhead speed at impact. However, as the mass and the first and second moments of the golf club all increase with increasing club length this is not necessarily the case. Reyes and Mittendorf (1999) considered the effect of club length in their double pendulum model of the swing of a driving champion. Keeping all other parameters fixed they found that increasing the club length from 119 (47 in) to 130 cm (51 in), an 8.5%

increase, resulted in the modelled clubhead speed increasing from 56.7 to 58.1 m s^{-1}, a 2.4% increase. These results are in good agreement with experimental measurements of clubhead speeds that they had taken with their golfer. They considered even longer clubs in their model and found that for a 152 cm (60 in) driver a clubhead speed of 59.9 m s^{-1} could be expected. However, as the shaft mass is held fixed in the model, this value for the clubhead speed is greater than would be expected. Also the model shows (Mittendorf and Reyes 1997) that increasing the club length, and keeping all other parameters fixed, will result in the clubhead lagging behind the golfers hands at impact. Therefore, in order to restore timing, a golfer switching to a longer club would need to adjust one or more of his or her swing parameters.

Werner and Greig (2000) measured clubhead speeds for numerous golfers swinging with drivers of varying lengths. Using this data, along with models for both the impact and the resulting golf ball trajectory, they determined that there was an optimum shaft length. The value they calculated was 128 cm (50.3 in) and was found to depend little on golfer size or skill level. However, the gain in drive distance that was calculated for the optimal length, as compared to a standard 117 cm shaft, was found to be only approximately 3 yards.

Decreasing the mass of the shaft will, in general, lead to increased clubhead speed. For a driver with a conventional steel shaft, approximately 25% of the kinetic energy that a golfer supplies to the club is spent on the shaft. Shafts are currently constructed from a variety of new lightweight materials. The major competition for the conventional steel shaft is the graphite shaft, which is a composite of carbon fibre and epoxy resin. A typical graphite shaft has a mass of approximately 90 g, compared to 120 g for a steel one. Pelz (1990) measured the total drive length for three professional golfers who were using both steel and graphite shafts. It was found that the average drive distance was approximately 3.0 yards longer when the graphite shafts were used. Werner and Greig (2000) used their swing model to model the effect of going from a 120 to a 50 g shaft. The result was an approximately 2% increase in clubhead speed and a gain of 3.3 yards in the drive distance.

Van Gheluwe *et al* (1990) looked at the effect that the use of graphite shafts has on swing kinematics. Subjects of various skill levels used clubs with both conventional steel and modern graphite shafts. It was found that the angular displacement of the upper arm and wrist-cock angle during the downswing did not display differences between the two types of shafts. They also did not find a significant difference between drive distances for the two shaft types.

4.1.2. Shaft flexibility. The shaft property that has received the most attention by researchers is the flexing of the shaft during the downswing. From measurements taken by Horwood (1994) and Butler and Winfield (1994), using strain gauges attached to club shafts, the general dynamic behaviour of a golf club shaft during the downswing has been determined. At the initiation of the downswing the shaft is bent backwards as a result of the inertia of the clubhead and the torque exerted by the golfer. As the downswing proceeds the shaft gradually straightens out and then bends forward during the final moments before impact. A popular view of golfers and many golf experts is that as the downswing proceeds the club springs back from its initial bent back state. This has led to the view that it is important to match the flexibility of the shaft to the swing speed of the golfer. It would be expected that for the right flexibility the springing back of the club should lead to greater clubhead speeds at impact. It is normal practice to recommend flexible shafts for weaker players and stiffer shafts for stronger players.

The major difficulty with this conventional view of shaft behaviour is that the natural frequency of oscillation of the clubhead and shaft system is much too high. Mather and Jowett (1998) calculated the fundamental frequency for the cantilever mode for various shaft and clubhead combinations. Their calculated values, which included the effect of centrifugal stiffening, for a clubhead speed of 45 m s^{-1}, ranged from 7.07 to 7.37 Hz. These values were in reasonable agreement with measured resonance values that they obtained by whirling shafts, with attached masses, about their butt ends at various frequencies. These relatively high natural frequencies would result in the club undergoing more than one oscillation over the time of a typical downswing, in disagreement with the measurements of Horwood, and Butler and Winfield. The damping provided by the golfers hands would, therefore, appear to be negating any possible effect that is due to the natural oscillation of the shaft during the downswing.

Milne and Davis (1992) modelled the swing of a golfer using a double pendulum model and included the flexibility, with centrifugal stiffening, of the shaft. The general simulated behaviour of the golf shaft during the downswing that they determined is shown in figure 9. As is shown in the figure, in the initial stages of the downswing the club is bent backwards. As was previously stated, this is due to the inertia of the clubhead and the torque applied by the golfer. The simulation finds that the shaft had its maximum backward deflection approximately 100 ms into the downswing. The shaft then gradually straightens out and as indicated in the figure, bends forward over the last approximately 40 ms of the downswing. Milne and Davis's model clearly shows that the bending forward of the shaft prior to impact is a result of the large centrifugal torque applied at its end by the clubhead. For a typical driver the centre of mass of the clubhead is located a few centimetres rearward of the shaft and due to the high clubhead speeds the effective weight of the clubhead is quite large, approximately 150 times its actual weight, at impact. The resulting centrifugal torque is found to be sufficient to explain the shaft's behaviour. Milne and Davis also measured the bending moment at three points along the shaft during the swing of three different golfers and found good agreement with simulated values. Although the forward deflection of the clubhead may be expected to increase the clubhead speed at impact, the simulation shows that, at the point of impact, the rate of increase of the clubhead deflection is found to be well past its maximum. Indeed, Milne and Davis

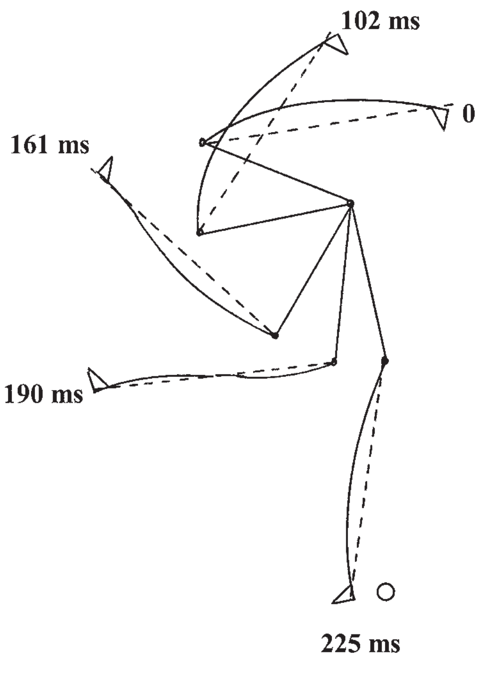

Figure 9. The simulated behaviour of a golf shaft during the downswing. Adapted from figure 1 in Milne and Davis (1992). The bending of the shaft is magnified by approximately 5.

found that for the flexible shaft model that was used, the launch speed of the golf ball was approximately 4% lower than that produced by a rigid shaft. The reason being that part of the energy transferred at impact went into exciting the flexible shaft.

Given Milne and Davis's model of the shaft behaviour, the primary effect of the flexibility of the shaft would be to alter the dynamic loft of the clubhead. The dynamic loft refers to the angle the clubface makes with the vertical at impact. The dynamic loft in general is not equal to the clubface loft, which is what is typically specified for a club. The clubface loft is the angle the clubface makes with the vertical when the sole of the clubhead is at rest on the ground. In the case of Milne and Davis's simulations, clubhead deflections, and therefore dynamic loft increases, were found to vary between 0.3° and 2.9° depending on the manner in which the hand couple is applied. Jorgensen (1994) found that for the professional golfer's swing that he analysed, the increase in the dynamic loft, due to the flexing of the shaft, was 3.3°. This increase in the dynamic loft would affect the launch parameters of the golf ball and, therefore, will have an effect on the trajectory and the drive distance. As will be discussed in section 4.2.1, the optimum dynamic loft for a driver decreases for increasing clubhead speeds. Therefore, the advice given to stronger golfers to use stiffer shafts, which would thereby reduce the dynamic loft of the clubhead, and for weaker golfers to use more flexible shafts, which would thereby increase the dynamic loft, may be valid. Experimentally, Pelz (1990) found that, from measurements made with professional golfers, increasing the shaft frequency, corresponding to increasing the stiffness, did result in increased average drives. However, it was not determined if this is as a result of the lowering of the dynamic loft or of reducing energy transfer to the shaft. One must be careful not to generalize, as measurements by Mather and Cooper (1994) found that the clubface deflection, or increase in dynamic loft, for a weak golfer using a regular shaft was 6.3° while a good golfer using the same shaft had a clubhead deflection of only 3.4°. It might have been expected that the greater clubhead speed generated by the good golfer would lead to a greater centrifugal torque and, therefore, a greater shaft deflection at impact. Mather and Cooper concluded that the amount of shaft deflection, and therefore head deflection, depends not only on the speed of the clubhead at impact but also on the time history of the input acceleration.

Mather and Jowett (2000) measured, using photogrammetry, the distribution of the radii of curvature for points along the length of given shafts at impact. They compared steel and graphite shafts as well as the effect of different clubheads and different golfers. They found that the distribution of the radii of curvature, and therefore the orientation of the clubhead at impact, was dependent on the shaft material, as well as on the distribution of stiffness along the shaft. Typically, shafts are tapered either continuously or in steps as one goes from the grip to the tip. However, they found that, in general, the performance characteristics are determined more by the design of the clubhead and less by the shaft parameters.

Researchers have also considered the deflection of the clubhead and shaft in other directions. As the centre of mass of the clubhead is also offset from the shaft by several centimetres towards the toe the centrifugal torque exerted by the clubhead will also cause the toe of the club to deflect down. Butler and Winfield (1994) found that, for the downswings that they measured, the amount that the toe of the clubhead was deflected downwards at impact ranged from 0.52 cm to as much as 5.44 cm. They also found that the shaft twisted by approximately 0.5° during the downswing, which resulted in the clubhead being slightly closed, or turned inwards towards the golfer, at impact. In general, it can be expected that the more flexible the shaft the more the toe of the clubhead will be deflected downwards and the more the clubhead will be closed at impact.

In addition to the behaviour of the shaft during the downswing, researchers have also considered the vibrations generated by the impact of the clubhead with the golf ball. These vibrations, which travel along the shaft, affect what golfers refer to as the ‘feel’ of the shot. Friswell *et al* (1996, 1998) measured the various frequency modes of a clamped steel shaft excited by a hammer blow at the clubhead. The fundamental frequency was found to be approximately 4.50 Hz, which is lower than the results of Mather and Jowett primarily due to their neglect of centrifugal loading. Friswell *et al*’s measurements of the various frequency modes were in good agreement with modelled behaviour, determined using finite element analysis for the shaft. The model shows that the asymmetrical head causes the natural frequencies to separate and couples the bending and torsional vibrations.

Ekstrom (1996) considered the ability of dampers, fitted inside the butt of golf shafts, to reduce the transmission of any shaft vibrations to the golfer. It was found that, in order to significantly reduce the vibrations, the frequency response of the damper needs to match that of the club. It was also found that only frequencies lower than approximately 50 Hz actually travel up a person’s arm farther than the wrist.

4.2. The clubhead

4.2.1. Optimum mass and loft. A primary characteristic of the clubhead of a driver that will affect the length of a drive is its mass. Neglecting the loft of the clubhead, and treating the collision between the clubhead and the golf ball as being one-dimensional, the launch speed of the golf ball will be given by

$$V_b = \frac{(1+e)V_c}{1+M_b/M_c}, \tag{6}$$

where V_b and M_b are the launch speed and the mass of the golf ball, V_c and M_c are the impact speed and mass of the clubhead, and e is the coefficient of restitution between the golf ball and the clubhead. In general, it would be expected that a more massive clubhead would be able to impart a greater launch speed to a golf ball, although from equation (6) it can be seen that there will be a limit to the launch speed of a golf ball, for in the case where $M_c \gg M_b$, V_b will equal $(1+e)V_c$. The benefit of a heavier clubhead will, however, be offset by the ability of a golfer to swing a heavier club as fast as a lighter one.

The theoretical effect of changing the clubhead mass can be determined from double pendulum swing models. Reyes and Mittendorf (1999) found, from their swing model, that reducing the clubhead mass from a conventional value of 191 g down to 170 g resulted in an 8.5% increase in clubhead speed. Using equation (6) this would correspond to approximately a 5.6% increase in the launch speed of the golf ball. Werner and Greig (2000), using their swing model, found that reducing the clubhead mass from 215 g down to 195 g resulted in the launch speed of the golf ball increasing by approximately 1.9%.

These effects of reducing the clubhead mass in the swing models are based on all the swing parameters except for the clubhead mass remaining fixed. Certainly, it would be expected that changing the clubhead mass, and therefore the mass and moments of the club, would affect the manner in which the golfer swings the club. Indeed, using the swing models, it is found that decreasing the clubhead mass, while keeping the other parameters fixed, will result in the hands leading the clubhead at impact. Therefore, it would be expected that a golfer would normally adapt his or her swing after switching to a lighter clubhead.

Daish (1972) presents experimental results of golfers using clubheads of different masses. Four golfers of varying abilities and strengths swung a club consisting of a conventional shaft carrying at its lower end brass discs, whose mass was varied from 100 to 350 g. Daish found that for all four subjects the speed of the clubhead at the bottom of the swing varied approximately as $M_c^{-0.19}$. Using this result in equation (6), Daish found that the optimum mass of the clubhead, irrespective of the golfer’s ability or strength, is approximately 200 g, in agreement with practice. Also, it is found that the effect of varying the clubhead mass is small, with an increase or decrease of 50 g from this optimum value resulting in a loss of launch speed of only approximately 0.5%. Daish's result and the relative insensitivity of the dependence of the launch speed of the golf ball on the clubhead mass is probably the reason why manufacturers opted to increase the size of the clubhead, thus compensating for miss-hits, rather than making the clubhead significantly lighter when using titanium.

Another characteristic of the clubhead of a driver that will affect the length of a drive is its loft, or the angle the clubface makes with the vertical. To determine what would be the optimum loft of a driver, the effect of the clubhead loft on all the launch parameters of the golf ball needs to be considered. In modelling the collision between the clubhead and the golf ball, researchers have in general made several simplifying approximations. The primary one is that the clubhead is treated as a free-body during the collision. Results from Cochran and Stobbs (1968) show that the shaft plays a small role during impact. Other approximations that are normally made are that the collision is taken to be instantaneous, with no change in the orientation of the clubhead during impact, that the collision is taken to occur at a single point, and that the golf ball is in a state of pure rolling when it leaves the clubface. It is also important to realize that the optimum lofts, determined by the various researchers, refer to the dynamic loft of the clubhead.

Since only central impacts are of concern when considering maximum drive distances, Penner (2001a) modelled the driver clubhead as a flat plate and determined the dependence of the golf ball's launch speed, launch angle, and launch spin on the loft of the clubhead. In general, it was found, that for a given clubhead speed, increasing the loft of the clubhead will result in a lower launch speed, a greater launch angle, and greater backspin for the golf ball. Penner then used the determined launch parameters along with suitable models of the trajectory and run to determine the length of both the carry and the drive. It was found that for a clubhead speed of 45 m s^{-1}, and a clubhead mass of 200 g, the dynamic loft which leads to the maximum carry, 209 yards, was 14.9°, while the dynamic loft which leads to the maximum drive, 232 yards, was 13.1°. The reason for the lower value for the drive is that lower impact angles lead to longer runs. The optimum value of 13.1° for the dynamic loft agrees well with the standard driver clubface loft, which is around 10°, when the magnitude of the angular deflection of the clubhead due to the flexing of the shaft is considered.

Penner also considered the effect that the clubhead speed has on the optimum loft. In general it was found that the greater the clubhead speed at impact, the lower the optimum loft. The primary reason being that the greater the clubhead speed at impact, the greater the amount of backspin imparted to the golf ball and the lower the optimal launch angle. For example, for a clubhead speed of 55 m s^{-1}, corresponding to a 260 yard drive, the optimum dynamic loft is only 10.7°. This result agrees, in general, with the practice of long hitting professional golfers using drivers with clubface lofts several degrees lower than the standard 10°. Overall, however, it was also found that the length of the drive is relatively insensitive to variations of the loft about the optimal value, with variations of 3° resulting in a loss in drive distance of approximately 5 yards.

Winfield and Tan (1994) presented a study on the optimization of both the loft and the swing elevation angle at impact. The swing elevation angle is the angle the path of the clubhead makes with the horizontal at impact. Many golfers will tee the ball up relatively high and forward in their stance so as to be able to hit the ball 'on the up' in order to increase drive distance. Winfield and Tan used a general, three-dimensional impact model between the clubhead and the golf ball to determine the launch parameters of the golf ball. A suitable aerodynamic model was then used to simulate ball flight and to determine the carry of the golf ball. In order to validate their simulation of the impact and the subsequent flight of the ball they compared the calculated carry of a golf ball struck by a driver, with a clubhead speed of $46\,\mathrm{m\,s^{-1}}$, a loft angle of 10.0°, and a swing elevation angle of 0°, with experimental results obtained with the use of a mechanical driver. The calculated value for the carry was 197.3 yards, which was in good agreement with the experimental value of 200.3 ± 3.6 yards.

Winfield and Tan then determined, by the use of a nonlinear optimization routine, the optimum loft and swing elevation angles for the given clubhead and a clubhead speed of $49\,\mathrm{m\,s^{-1}}$. For a swing elevation angle of 0° the optimum loft of the clubhead, in terms of the carry distance, was found from their model to be 12.2° resulting in a carry distance of 220 yards. Increasing the swing elevation angle, in general, results in a lower optimum loft. For example, for a swing elevation angle of 5° the optimum loft angle was found to be 10.2° resulting in a carry of 230 yards.

4.2.2. Clubface flexibility. A characteristic of the clubface, which has, in recent years, captured the attention of the golfing community, is the 'spring' or 'trampoline' effect. The clubhead is normally modelled as being rigid with all the energy lost during impact occurring with the deformation of the golf ball. Indeed, it might be expected that the less the clubface deforms, the less the overall energy loss, and, therefore, the greater the rebound speed of the golf ball. This is true, in general, and a golf ball will rebound at a greater speed from a very rigid surface than from a very flexible one. However, if the natural frequency of the flexing of the clubface matches that of the golf ball's compression and recovery rate during impact, the clubface can actually add a slight push to the golf ball during the rebound stage. In terms of energy, the deformation of the clubface during the collision is such that less energy is lost during its deformation than in that part of the ball's deformation, which it replaces. The ability to use the 'trampoline' effect has risen both with the use of larger clubheads and with the use of titanium. Normally, the natural compression and recovery frequency of a golf ball, which is approximately 1000 Hz, is much lower than the frequency of the fundamental flexing mode of a clubface. However, the strength to weight advantage of titanium allows clubfaces to be made much thinner and clubheads to be made larger. A larger and thinner clubface can result in its natural frequency decreasing down to values approaching that of a golf ball.

Several researchers have considered the trampoline effect and have presented models and experimental results of the dependence of the coefficient of restitution for collisions between golf balls and targets. Cochran (1999) considered the collision between a golf ball and a thin plate mounted rigidly at its edges. A two-parameter golf ball model was used along with a two-parameter model of the plate, which is treated as an effective mass attached to a parallel combination of a linear spring and a dashpot. In general, the spring constant associated with the plate will depend on its geometry, mass and material properties. Cochran showed that, if the natural frequency of the flexing and recovery of the plate was near that of the golf balls compression and recovery, the coefficient of restitution could be raised by as much as 12%, depending on the amount of damping that is included in the model. This corresponds to an increase of approximately 5% in ball launch speed for a typical drive.

Johnson and Hubball (1999) also modelled the collision between a golf ball and a flexible clubface. They used Johnson and Lieberman's five-parameter model for the golf ball and a linear two-mass/one-spring representation of the target. They compared their modelled results with experimental results, which involved firing golf balls at freestanding titanium plates of various thicknesses. They found that, for both the modelled and the experimental results, as the thickness of the titanium plate was reduced from 6.35 mm down to 2.54 mm, the coefficient of restitution increased by approximately 15%.

Yamaguchi and Iwatsubo (1999) present results showing the dependence of the coefficient of restitution on the values used for the spring constant and the damping constant in their model of a clubface. They, like Cochran, found that the maximum value of the coefficient of restitution occurred when the natural frequencies of the clubface and golf ball matched. Their modelled results agreed with the experimental measurements they made by firing a simulated clubhead at golf balls. In addition, they also presented experimental results showing how the natural frequency of the clubface depends on clubhead volume and material. While the typically smaller 150 cc stainless steel clubheads have a natural frequency of about 1800 Hz, the larger 250–300 cc titanium clubheads are found to have a natural frequency of approximately 1200 Hz. The natural frequency of golf balls was also found to vary, ranging from 800 Hz for soft two-piece balls up to 1300 Hz for hard ones. This indicates that to optimize the trampoline effect, the natural frequency of the clubhead would need to be matched to that of the golf ball.

4.2.3. Clubface roughness. Another feature of the clubface that researchers have looked into is the roughness of its surface. It would be expected that the rougher the surface of the clubface, the greater the amount of spin imparted to a golf ball during impact, and this is the reasoning behind the grooved and sandblasted faces of clubheads. However, as Cochran and Stobbs (1968) discuss, this is not always so. They indicate that although the roughness of the clubface will determine how quickly the ball stops sliding and begins to roll along the clubface during the impact, it will have little effect on the final spin of the rebounding golf ball. To demonstrate this, they had pairs of 5-irons, 7-irons and 9-irons constructed. In each pair one club had a normal grooved and sandblasted surface, whereas the other had a completely smooth face. Measurements are presented for golf shots hit by a professional golfer and it is found that both clubfaces impart approximately the same spin to the golf ball and result in the same carry and run.

Chou *et al* (1994) present measured and modelled results for the dependence of the spin rate on loft angle for a one-piece golf ball projected against a steel barrier with both a smooth and a rough surface. For loft angles below 40°, the results show that the spin rate acquired by the golf ball for both surfaces is approximately the same. This agrees with the general results of Cochran and Stobbs. However, at higher loft angles, such as would be found with wedges, differences did occur. For example, at a loft of 50°, and an impact speed of 45.9 m s^{-1}, it was found that the rougher surface imparted approximately 250 rps of backspin to the golf ball compared to only 150 rps for the smoother surface. Their model of the collision shows that for the higher lofted impacts against the smoother surface the golf ball does not reach a state of pure rolling and slides throughout the collision with the barrier.

Gobush (1996b) presents similar sets of measurements for golf balls projected against rough steel, smooth steel and Teflon plates. In these measurements both three-piece and two-piece golf balls were considered and loft angles were varied from 0° to 50°. The spins imparted to the soft covered three-piece ball for both the rough and the smooth steel surfaces were found to be the same over the full range of tested loft angles. For the Teflon plate the spin rates matched the steel plates for lofts up to 20° but then dropped off sharply after that. The results for the hard covered two-piece ball were much less predictable. For example, for loft angles greater than approximately 20°, the smooth steel plate imparted the greatest backspin to the golf ball, followed by the rough steel plate, and then the Teflon plate. In general, it was concluded that the ball type has much more effect on the spin imparted to the golf ball than the roughness of the clubface.

The result, where smoother surfaces can under certain conditions impart greater spin rates, is further highlighted by Lieberman (1990) who presents results from a study that involved firing golf balls against barriers with both dry and grassy surfaces. Surprisingly, it was found that for lofts less than approximately 35° for two-piece balls, and 45° for three-piece balls, the grassy surface resulted in spin rates of the order of 10% greater than for the dry surface. For the higher loft angles, the dry surface resulted in significantly greater spin rates than what was found for the grassy surface.

Ekstrom (1999) presents results showing how the roughness of the stainless steel clubface inserts affects the coefficient of sliding friction. The experimental technique involved driving various golf ball types down between two fixed inserts. Three pairs of sandblasted stainless steel inserts were tested with a RMS surface roughness of 25 μin, 100 μin and 150 μin, respectively. Both the normal and tangential forces exerted by the golf balls on the inserts were measured with strain gauges and from these the coefficient of sliding friction was calculated. It was found that although the coefficient of sliding friction did increase as the roughness went from 25 to 100 μin, it did not increase any further for the 150 μin surface. This would indicate that there would be a limit to the benefit of roughening the surface of clubfaces.

4.2.4. Off-centre impacts. In order to maximize the distance of their drives, golfers will wish to strike the ball at the centre of the clubface. Unfortunately, golfers will frequently strike the ball off-centre. There are several characteristics of the clubhead that affect its performance with respect to off-centre impacts. One of these, in particular, is the shape of the clubface of a driver.

Off-centre impacts will cause a clubhead to rotate about its centre of mass, which will in turn cause the clubface to move along the surface of the golf ball, thereby imparting spin. This imparting of spin to a golf ball as a result of the rotation of the clubhead during the collision is referred to in golf as the 'gear effect'. In the case of miss-hits in the lateral direction, sidespin will be added to the golf ball. If the golf ball is struck near the toe of the clubhead, the rotation of the clubhead and the resulting sidespin will cause the ball to hook and to land, in the case of a right-handed golfer, to the left of the fairway. Conversely, a heel shot will result in sidespin that will cause the ball to slice and to land to the right of the fairway. The lateral profile of the clubface of a driver is, therefore, slightly convex so that the golf ball will be launched, in the case of off-centre impacts in the lateral direction, in a direction pointing away from the centre of the fairway. The gear effect will then result in the golf ball curving back towards the middle of the fairway.

For impacts above or below the centre of the clubface, the clubhead will again rotate about its centre of mass and will in this case affect the amount of backspin with which the golf ball is launched. Impacts below the centre will result in the golf ball being launched with more backspin while those above the centre will result in the golf ball being launched with less backspin. The greater the amount of backspin with which a golf ball is launched the lower the optimum launch angle, with regard to drive distance. To maximize drive distances the loft of the clubface should, therefore, decrease as one moves vertically down the clubhead. So to account for miss-hits in both the lateral and vertical directions the clubheads of drivers are normally manufactured so as to have the clubface bulging slightly outwards.

Several researchers have modelled the general three-dimensional impact between a clubhead and a golf ball in order to determine the optimum shape or the optimum curvature of the clubface. Penner (2001b) modelled off-centre impacts in the lateral direction, treating the clubhead as a free-body during the collision and with the impact occurring at a single point on the clubface. By using models for the golf ball trajectory and run, Penner was able to determine, for a specific impact point, the radius of curvature of the clubface that would result in the golf ball stopping in the middle of the fairway. Figure 10 shows the dependence of the trajectory on the radius of curvature for an impact point 2.0 cm from the clubface centre. As shown in the figure, for a clubhead speed of 45 m s^{-1}, the optimum radius of curvature for the given clubhead model is 21.5 cm. This value is found to be in reasonable agreement with current clubhead design and empirical results. For example, Maltby (1990) used a mechanical golfer to determine empirically that the optimum radius of curvature in the lateral direction, for

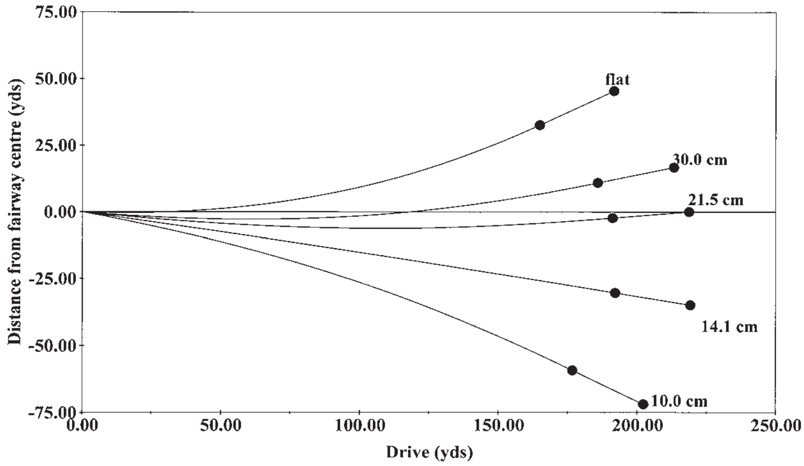

Figure 10. Overhead view showing the dependence of the trajectory and run of a golf ball, for various radii of curvature of the clubface for an impact point 2.0 cm from the clubface centre. Adapted from figure 8 in Penner (2001b).

the clubface of a driver, ranges from approximately 20.3 to 27.9 cm. In addition, Penner found that increasing the clubhead mass or volume, and thereby increasing the clubhead's moment of inertia, would reduce the gear effect along with the amount of curvature required for the clubface. It was also found that increasing the impact speed of the clubhead would increase the gear effect and therefore, in general, would require more curvature for the clubface.

Winfield and Tan (1996) have also presented a model of the general three-dimensional impact between a clubhead and a golf ball. The clubhead is again treated as a free-body, with the impact taken to act at a single point. The shape of the clubface is taken to be ellipsoid and off-centre impacts in both the lateral and vertical directions are considered in this model. The models of the impact as well as the ball trajectory that were used were tested by taking experimental measurements of the landing positions of golf balls struck by a golfing robot off the centre, heel and toe of a clubhead of a standard driver. The experimental results agree quite well with the modelled results. As an example, for a heel shot 1.59 cm from the clubface centre, the model shows that a golf ball loses approximately 12 yards, on the carry, and lands approximately 8 yards from the centre of the fairway. Experimentally, the loss in distance for the same drive was approximately 12 ± 4 yards and the golf ball landed 11 ± 3 yards from the fairway centre.

Winfield and Tan then used an optimization routine to calculate the radii of curvature of the clubface that would minimize the dispersion of golf ball landing positions about the centre of the fairway. They found that, in general, to decrease the dispersion of the landing positions of the golf ball would require a less convex shape than is normally found with drivers. For the clubhead that they considered, and at a clubhead speed of $46\,\mathrm{m\,s^{-1}}$, the optimal radius of curvature in the lateral direction was determined to be 59.8 cm. Part of the reason for this relatively high value is that the run of the golf ball is neglected in the model. They also found that the curvature of the clubface should increase with the distance between the centre of mass of the clubhead and the clubface centre. This is expected as the farther the centre of mass is from the clubface, while keeping the inertia properties of the clubhead fixed, the greater the gear effect will be.

Two other key characteristics of a golf clubhead, with regard to off-centre impacts, are its moments of inertia and the location of its centre of gravity. These will affect the amount that the clubhead will twist during off-centre impacts and, thereby, the amount of spin imparted to the launched golf ball. Certainly an advantage of the modern day hollow metal driver clubhead over the traditional solid wooden one is its larger moments of inertia. Currently metal clubheads are manufactured with increasingly greater volumes to even further increase the moments of inertia. With modern day iron clubheads perimeter weighting is the norm, with more and more of the clubhead mass being placed along the perimeter of the clubhead.

Iwatsubo *et al* (2000) specifically consider the effect that the moments of inertia of the clubhead of a driver and the location of its centre of gravity have on the launch parameters of a golf ball, in the case of off-centre impacts. Finite element models were constructed for both the golf ball and for various driver clubheads and impacts were simulated for various points on the clubface. As expected, the larger the moment of inertia of the clubhead about the vertical axis the less the amount of sidespin imparted to golf balls struck off the centre of the clubface. As an example, for an impact 10 mm towards the toe, the sidespin imparted for a given clubhead was approximately 3.1 rps, while for the same shot with a clubhead with a moment of inertia about the vertical axis approximately 19% greater, the sidespin imparted was approximately 2.3 rps. It was also found that the closer the centre of gravity of the clubhead was to the clubface centre the less the sidespin imparted. This is expected, as the magnitude of the gear effect would decrease as the centre of gravity moved forward.

Iwatsubo *et al* also considered the effect that the moment of inertia of the clubhead about the lateral axis, and the position of the centre of gravity, has on the backspin imparted to golf balls impacted above or below the clubface centre. Surprisingly, it was found that the greater this component of the moment of inertia was, for the different clubhead models, the greater the amount of backspin imparted. It is suggested that this result may be due to the variation in the other parameters used for the different clubhead models. It was also found that, in general, the nearer the centre of gravity is to the clubface centre the greater the imparted backspin for off-centre impacts.

Although the modern day metal clubhead of a driver leads to better performance with regard to off-centre impacts, as compared to the traditional solid wooden clubhead, it is the perception of many experienced golfers that the metal clubhead does not have as good a ‘feel’ as the older drivers. In line with this, Hocknell *et al* (1996) compared the differences between the generated vibrations and the sound produced by various clubhead and ball impacts. One of the notable differences that they found between the metal and wooden clubheads was the sound they produced. The hollow metal clubhead produced sound in the relatively high frequency range, between 5 and 11 kHz, during impact. The wooden clubhead did not generate such high frequencies. Although the negative perception that golfers have towards the higher pitched sound is psychological it is considered important by golf club manufacturers.

Several researchers have considered the effect of off-centre impacts with non-driver shots. Knowles *et al* (1998) modelled the impact between a two-piece golf ball, using Johnson and Lieberman’s five-parameter model, and a 4-iron. Impacts were simulated for various impact points on the clubface and results were compared with experimental measurements taken with professional golfers. The agreement was found to be reasonable. For example, for test points on the heel of the club the simulated launch speed for the golf ball impacted by the clubhead, which was moving at $40\,\mathrm{m\,s^{-1}}$, was $51\,\mathrm{m\,s^{-1}}$ as compared to $54\,\mathrm{m\,s^{-1}}$ for a centre hit. In the case of the experimental results, normalized to the same clubhead speed used in the simulation, the average launch speed of the golf ball for a heel shot was $51.6\,\mathrm{m\,s^{-1}}$ as compared to $56.9\,\mathrm{m\,s^{-1}}$ for a centre impact.

Werner and Greig (2000) looked at the benefit of using a putter with a large moment of inertia about the vertical axis. Many modern day putters have this feature, accomplished by placing most of the clubhead mass at the heel and the toe or far back of the putter face. They compared the effects of off-centre hits using such a putter with a more conventional one, which had a uniform mass distribution. As an example, they found that for the high moment of inertia putter and a 4.6 m (15 ft) putt, an off-centre hit that is 2.54 cm towards the toe will result in the putt being 33 cm short compared with 53 cm short for the conventional putter. However, overall they found that the variations in stopping points between the two putters are much smaller than the variations caused by golfer errors in clubhead alignment and speed.

4.3. Club design

Researchers have examined the effect that club design has on the swing of a golfer. Both club matching and recent innovative designs will be considered in this section.

In general, a golfer would want the different clubs in their set to 'feel' the same in identical swings. This would result in greater consistency. For the clubs to 'feel' the same equation (6) shows that they would need to have the same mass and the same first and second moments about the wrist-cock axis. However, in a typical set of golf clubs the length of the shafts vary as most golfers want more control, and thereby shorter shafts, for the shorter shots. The three dynamic parameters do, therefore, vary from club to club.

Conventionally golf club manufacturers produce sets of clubs wherein the first moment about a particular point on the shaft, normally 30.5 cm (12 in) from the grip end, is the same for each club. This is referred to as swing-weight matching. There seems to be no particular reason for doing this other than historical. However, it does turn out that swing-weight matching is a compromise between matching the first and second moments of the club. Swing-weight matching results in the first moment of the club about the grip increasing as one moves from the 1-iron to the 9-iron and the second moment decreasing.

Jorgensen (1994) presents a club matching technique in which all three dynamic parameters of the clubs are the same. This method involves the judicious placing of two slugs of different masses within the shaft of each club, as well as adjustments to the clubhead masses.

Budney and Bellow (1982) determined how the clubhead mass should vary within a set so that the resulting circumferential force applied at the grip during the downswing would have the same maximum value for each club. They refer to this as dynamic modelling and found that for the shorter irons the clubheads should have a slightly smaller mass, less than 3.5%, than found with a conventionally matched set. However, it is also shown that if a golfer preferred to swing the short irons considerably slower than the longer clubs even greater clubhead mass than that prescribed by conventional matching would be required.

Mather (2000) looked at how to better match clubs to the swings of amateur golfers. Amateurs are found (Cooper and Mather 1994) to accelerate their golf club at a much greater rate early in the downswing as compared to professional players. This leads to the club exerting greater forces on the amateur during these crucial moments and, therefore, results in less control. Mather discusses the effect of adding mass to the grip end of the club. This will increase the moment of inertia about the hub during the initial stages of the downswing but during the final stages of the downswing the moment of inertia of the club about the wrists will be unaffected.

Sprigings and Neal (2001) examined the effect that moving a portion of the mass from the grip end of a golf club to a position farther down the golf shaft has on clubhead speed. They concluded that the effect was dependent on the nature of the torque applied to the club. In the case where only gravitational torque is acting on the club during the downswing the optimal location of the given added mass was approximately halfway down the shaft. However, in the more realistic case where torque is also applied at the grip, shifting any of the grip mass down the shaft results in lower clubhead speeds at impact.

Nishizawa *et al* (2000) examined the effect of adding water to the inside of the shaft. At the onset of the downswing the water will be at the grip end, but as the downswing proceeds the water will move towards the clubhead. They simulated the effect of such a club using a double swing model and found that the angular momentum of the club was 44% greater at impact than for a conventional club.

5. Conclusion

As stated in the introduction, the primary purpose behind the theories, models and experiments that have been reviewed in this paper is to determine ways of improving the performance of both skilled and unskilled golfers.

For many of the studies that have focused on the golf swing, this has corresponded to determining ways of increasing the clubhead speed at impact and thereby increasing the length of the golf shot. The double pendulum swing model shows the relative benefits of increasing the hub torque, the backswing angle, and the wrist-cock angle. The difficulty for the golfer is that changing one of these swing parameters will normally require a change in another if the club is to be in the correct position at impact. Also, and this is critical, unless the timing of the swing is correct, any possible benefit of increasing one of the above swing parameters will be lost.

One of the defining differences between the swings of skilled and unskilled golfers that measurements have indicated is the delay in the release of the golf club. Experimental results and the double pendulum swing model clearly show that poor timing, associated with the early release found with the swings of unskilled golfers, results in the loss of clubhead speed. However, as the downswing only lasts approximately 200 ms it would be difficult for the unskilled golfer to conscientiously apply a further delay in the release without completely throwing off the timing of the swing. Also, the experimentally determined couples that are applied at the pivot points are found to be significantly different from what is normally used in the two-stage swing model. Therefore, incorporation of more realistic couples in swing models, whether they be two-link or three-link, would be required in order to fully understand how skilled golfers delay the release of the golf club.

The double pendulum swing model also clearly shows how energy is transferred from the golfer to the club. In the future, triple-link models, such as Turner and Hill's, will hopefully also give insight into the correct timing of the rotation of the hips, the trunk, and the shoulders of the golfer, in order to optimize their energy transfer.

Research into the behaviour of a golf ball has again primarily been focused on ways in which the performance of golfers can be improved. A golfer will normally benefit from a golf ball that will give maximum distance, as well as maximum control around the green. These two characteristics are typically in direct opposition, for in order to maximize distance the spin of the golf ball needs to be constrained while for shots around the green a golfer wishes to maximize the amount of spin. Many of the studies have focused on the behaviour of different golf ball constructions, primarily two-piece and three-piece balls, during impact with fixed barriers. In general, two-piece balls are found to have greater coefficients of restitution and to acquire less spin during impact. These balls will, therefore, travel further. Three-piece balls are found to acquire more spin. However, research has demonstrated that the critical design parameters of a golf ball has not so much to do with whether the ball is two-piece, three-piece, or even multilayered but on its flexural properties and the hardness of its cover. This research has and will lead to new ball designs where both reduced spin during drives and increased spin during shots around the green will be available with one ball.

Studies of golf ball aerodynamics have led to increased understanding of the effect of different dimple patterns and, in turn, to improved ball flight characteristics. Another benefit of obtaining more accurate models of aerodynamic forces is that this allows for more intensive and quicker tests of golf balls under various launch conditions. This not only allows for better matching of golf balls to players' abilities but also provides an efficient way for governing bodies to determine if golf balls conform to the rules.

Research into the design of golf clubs has primarily centred on increasing drive length and reducing the effect of off-centre impacts. The effect of shaft mass and length as well as clubhead mass and loft on drive distance has been modelled and the results match reasonably well with measurements. However, further research is required to determine how these club characteristics would be optimized for particular swing patterns and golfer abilities.

The effect of shaft flex is still a topic of debate in the golf community. Many golf experts and manufacturers still tout the benefit of weaker players using more flexible shafts as a means of increasing clubhead speed. The modelling and measurements that have been done show that this advice is at best very questionable. The more likely reason for manufacturers providing a range of shaft flexes is to compensate for their clubheads, which are in general manufactured with the same clubface loft.

The springing back of the clubface also requires further modelling and measurements. Experiments involving actual clubheads as well as analysis of off-centre impacts are required before the magnitude of the benefit of using such a clubface is known.

Overall, there are limits to the performance of golf balls and golf clubs. In their quest to produce better golf equipment, researchers and manufacturers are constrained by the rules of golf as laid out by the Royal and Ancient Golf Club of St Andrews and the United States Golf Association. Golf balls and golf clubs, specifically the driver, are currently at or near the performance limits that have been set. In the future it can be expected that research will be focused on better matching of golf balls and golf clubs to a golfer's swing characteristics and skill level.

Finally, although it may be comforting for the average golfer to know that a multitude of researchers are working relentlessly to improve the performance of golfers, the game of golf is not easily tamed. Hopefully, in the future, physics will be able to explain why a golfer will consistently put his or her golf ball into the water hazard at a particular hole on their local golf course or why the probability of making a putt seems to be inversely proportional to the importance the golfer places on it!

References

Achenbach E 1972 *J. Fluid Mech.* **54** 565

Alessandrini S M 1995 *SIAM Rev.* **37** 423–7

Aoki K, Nakayama Y, Hayasida T, Yamaguti N and Sugiura M 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 446–56

Aoyama S 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 457–63

Bearman P W and Harvey J K 1976 *Aeronaut. Q.* **27** 112–22

Budney D R and Bellow D G 1979 *Res. Q. Exer. Sports* **50** 171–9

Budney D R and Bellow D G 1982 *Res. Q. Exer. Sports* **53** 185–92

Budney D R and Bellow D G 1990 *Science and Golf I* ed Cochran (London: E & FN Spon) pp 30–5

Butler J H and Winfield D C 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 259–64

Chou P C, Liang D, Yang J and Gobush W 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 296–301

Cochran A and Stobbs J 1968 *The Search for the Perfect Swing* (New York: Lippincott)

Cochran A J (ed) 1990 *Science and Golf I: Proc. First World Scientific Congress of Golf* (London: E & FN Spon)

Cochran A J and Farrally M R (ed) 1994 *Science and Golf II: Proc. 1994 World Scientific Congress of Golf* (London: E & FN Spon)
Cochran A J 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 486–92
Cooper M A J and Mather J S B 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 65–70
Daish C B 1972 *The Physics of Ball Games* (London: English University Press)
Dillman C J and Lange G W 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 3–13
Ekstrom E A 1996 *The Engineering of Sport* ed Haake (Rotterdam: Balkema) 315–22
Ekstrom E A 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) 510–18
Erlichson H 1983 *Am. J. Phys.* **51** 357–61
Farrally M R and Cochran A J (ed) 1999 *Science and Golf III: Proc. 1998 World Scientific Congress of Golf* (Leeds: Human Kinetics)
Fox W and McDonald A T 1992 *Introduction to Fluid Mechanics* 4th edn (New York: Wiley) p 445
Friswell M I, Smart M G and Hamblyn S M 1996 *The Engineering of Sport* ed Haake (Rotterdam: Balkema) pp 323–31
Friswell M I, Mottereshead J E and Smart M G 1998 *Sports Eng.* 41–50
Gobush W 1990 *Science and Golf I* ed Cochran and Farrally (London: E & FN Spon) pp 219–24
Gobush W 1996a *The Engineering of Sport* ed Haake (Rotterdam: Balkema) 193–4
Gobush W 1996b *Golf the Scientific Way* ed Cochran (Hemel Hempstead: Aston) pp 141–5
Haake S J 1991 *Applied Solid Mechanics 4* ed Ponter and Cocks (Amsterdam: Elsevier) pp 72–89
Haake S J 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 431–36
Hocknell A, Jones R and Rothberg S 1996 *The Engineering of Sport* ed Haake (Rotterdam: Balkema) pp 333–7
Hocknell A, Jones R and Rothberg S J 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 526–34
Holmes B W 1991 *Am. J. Phys.* **59** 129–36
Horwood J G P 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) 247–58
Hosea T M, Gatt C J, Galli K M, Langrana N A and Zawadsky J P 1990 *Science and Golf I* ed Cochran (London: E & FN Spon) pp 43–8
Hubbard M and Alaways L W 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 429–39
Iwatsubo T, Kawamura S, Miyamoto K and Yamaguchi T 2000 *Sports Eng.* **3** 195–204
Johnson S H and Lieberman B B 1994a *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 309–14
Johnson S H and Lieberman B B 1994b *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 315–20
Johnson S H and Lieberman B B 1996 *The Engineering of Sport* ed Haake (Rotterdam: Balkema) pp 251–6
Johnson S H and Hubball J E 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 493–9
Johnson S H and Ekstrom E A 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 519–25
Jorgensen T 1970 *Am. J. Phys.* **38** 644–51
Jorgensen T P 1994 *The Physics of Golf* (New York: Springer)
Kaneko Y and Sato F 2000 *The Engineering of Sport: Research, Development and Innovation* ed Subric and Haake (London: Blackwell) pp 469–76
Knowles S, Mather J S B and Jowett S 1998 *The Engineering of Sport: Design and Development* ed Haake (Cambridge: Cambridge University Press) pp 333–42
Koenig G, Tamres M and Mann R W 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 40–5
Lampsa M A 1975 *ASME 97 (Series G)* pp 362–7
Lemons L D, Stanczak M B and Beasley D 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 423–8
Lieberman B B 1990 *Science and Golf I* ed Cochran (London: E & FN Spon) pp 225–30
Lorensen W E and Yamrom B 1992 *IEEE Comput. Graphics Appl.* **12** 35–44
MacDonald W M and Hanzely S 1991 *Am. J. Phys.* **59** 213–18
McPhee J J and Andrews G C 1988 *Am. J. Phys.* **56** 933–9
McTeigue M, Lamb S R, Mottram R and Pirozzolo F 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) 50–8
Mahoney J F 1982 *Res. Q. Exer. Sport.* **53** 165–71
Maltby R 1990 *Golf Club Design, Fitting, Alterations and Repair* (Newerk: Maltby)
Mather J S B and Cooper M A J 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 271–7
Mather J S B and Immohr J 1996 *The Engineering of Sport* ed Haake (Rotterdam: Balkema) pp 221–8
Mather J S B and Jowett S 1998 *The Engineering of Sport: Design and Development* ed Haake (London: Blackwell) pp 515–22
Mather J S B 2000 *The Engineering of Sport: Research, Development, and Innovation* ed Subic and Haake (London: Blackwell) pp 61–8

Mather J S B and Jowett S 2000 *The Engineering of Sport: Research, Development, and Innovation* ed Subic and Haake (London: Blackwell) pp 77–86
Milburn P D 1982 *Med. Sci. Sports Exercise* **14** 60–4
Milne R D and Davis J P 1992 *J. Biomechanics* **25** 975–83
Mittendorf A and Reyes M G 1997 http://www.Golfphysics.com
Miura K 2001 *Sports Eng.* **4** 75–86
Neal R J and Wilson B D 1985 *Int. J. Sport Biomechanics* **1** 221–32
Nishizawa S S, Sugiyama T and Watanabe K 2000 *The Engineering of Sport: Research Development, and Innovation* ed Subic and Haake (London: Blackwell) pp 69–76
Pelz D 1990 *Science and Golf I* ed Cochran (London: E & FN Spon) pp 264–9
Penner A R 2001a *Am. J. Phys.* **69** 563–8
Penner A R 2001b *Am. J. Phys.* **69** 1073–81
Penner A R 2002a *Can. J. Phys.* **80** 83–96
Penner A R 2002b *Can. J. Phys.* **80** 931–40
Pickering W M 1998 *The Engineering of Sport: Design and Development* ed Haake (London: Blackwell) pp 353–60
Pickering W M and Vickers G T 1999 *Sports Eng.* **2** 161–72
Reyes M G and Mittendorf A 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 13–19
Roberts J R, Jones R and Rothberg S J 2001 *Sports Eng.* **4** 191–203
Robinson R L 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 84–90
Smits A J and Smith D R 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 341–7
Sprigings E J and Neal R J 2001 *Sports Eng.* **4** 15–21
Stanczak M B, Lemons L D, Beasley D E and Liburdy J A 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 440–5
Sullivan M J and Melvin T 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 334–9
Tavares G, Shannon K and Melvin T 1999a *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 464–72
Tavares G, Sullivan M and Nesbitt D 1999b *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 473–80
Turner A B and Hills N J 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 3–12
Ujihashi S 1994 *Science and Golf II* ed Cochran and Farrally (London: E & FN Spon) pp 302–8
Van Gheluwe B, Deporte E and Balleeer K 1990 *Science and Golf I* ed Cochran (London: E & FN Spon) pp 258–63
Vaughan C L 1979 *Biomechanics VII-B* ed A Morecki, K Fidelus, K Kedzior and A Wits (Baltimore: University Park Press) pp 325–31
Watanabe K, Kuroki S, Hokari M and Nishizawa S 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 29–39
Werner F D and Greig R C 2000 *How Golf Clubs Really Work and How to Optimize Their Design* (Jackson: Origin Inc.)
Williams D 1959 *Q. J. Mech. Appl. Math.* **XII** 387–92
Winfield D C and Tan T E 1994 *Comput. Struct.* **53** 19–25
Winfield D C and Tan T E 1996 *Comput. Struct.* **58** 1217–24
Yamaguchi T and Iwatsubo T 1999 *Science and Golf III* ed Farrally and Cochran (Leeds: Human Kinetics) pp 500–9